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Novel Technique for Obtaining Double-Layer Tensegrity Grids

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SUMMARY: Double-layer tensegrity grids (DLTGs) may be defined as tensegrity spatial systems containing two parallel horizontal networks of members in tension forming the top and bottom layers, whose nodes are linked by vertical and/or inclined bracing members in compression and/or tension.

In this paper, a new approach is described. Conventional double-layer grids (DLGs) are composed of three layers: top, bottom and bracing members. This paper shows new rules for generating original DLGs following a recent methodology for their composition, from the mosaic of the bracing members and additional laws. Finally, from them, a new technique, known as Rot-Umbela manipulation, is applied to obtain their tensegrity form, opening and endless catalogue of DLTGs.

Key Words: Tensegrity, Structures, Double-Layer, Grids, Design, Tessellations, Rot-Umbela Manipulation

1. INTRODUCTION

Tensegrity is a principle based on self-stressed and auto-stable structures composed by isolated components in compression inside a net of continuous tension, in such a way that the compressed members (usually bars or struts) do not touch each other and the pre-stressed tensioned members (usually wires or even tensile membranes) delineate the system spatially [1]. Based on this concept, double-layer tensegrity grids (DLTGs) are defined as tensegrity spatial systems containing two parallel horizontal networks of members in tension (top and bottom chords), whose nodes are linked by vertical and/or inclined bracing members in compression and tension.

These kinds of structures are being taken into account in the last years with increasing frequency for the construction of several canopies, roofs, covers and even bridges (like the Kurilpa Bridge in Brisbane, Australia). Even though some of them could not be considered as pure tensegrity structures, there is a rising sensibility to their application with engineering and architectural purposes. It is remarkable that researches on Tensegrity have not decreased. In fact, for all the publications about Tensegrity since the 70s, 83% of them have been

published during the last decade (according to the databases of Scopus and Web of Knowledge).

1.1. Organization of the paper

Firstly, relevant precedents on DLTGs over the last few years, concerned with this work, will be presented briefly as an introduction to the topic. Then, the basic two methodologies used at the moment for the generation of DLTGs will be exposed, based on composition and decomposition techniques. After that, a new approach will be proposed, parting from conventional double-layer grids (DLGs) and applying to them a new kind of operation, denominated Rot-Umbela manipulation. It will be explained that some of the current tensegrity grids could be obtained by Rot-Umbela manipulations. Finally, some notes about other future proposals, analysis and conclusions are presented as part of a research with larger implications.

1.2. Relevant precedents on DLTGs

Since the controversial discovery of Tensegrity [2] in the 1940s–50s, many configurations of tensegrity grids have been proposed, starting with the works of their discoverers: Fuller, Snelson and Emmerich. In the 70s, Pugh [3] proposed some tensegrity nets,

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although it was not until the next decade when Hanaor [4] and Motro [5] took a more structural and mechanical approach and studied the form-finding, resistance and stability of some DLTGs. The former experienced basically with the juxtaposition of tensegrity prisms, avoiding contacts between struts; meanwhile, the latter studied the same tensegrity pyramids (mainly the same half-cuboctahedrons showed by Emmerich [6] in his first patent) but for planar grids, by means of joining the ends of some struts. Emmerich [7] also published a complete book, where a chapter was dedicated exclusively to “self-stressed planar nets” from a geometrical point of view.

Following their steps, some other studies were undertaken. Since 1996, Wang B.B., one of the most prolific authors, analyzed thoroughly the combinations of modules to generate several types of DLTGs, depending on many different factors [8].

At the same time, Kono et al. [9] experimented with a single kind of tensegrity grid, based on the use of tripods or truncated pyramids of 3 bars each, somehow similar to design #4 of Emmerich’s patent [6] (although with some vertex-to-vertex connections) (Fig 1.a) and Snelson’s abandoned patent [10] (Fig 1.b).

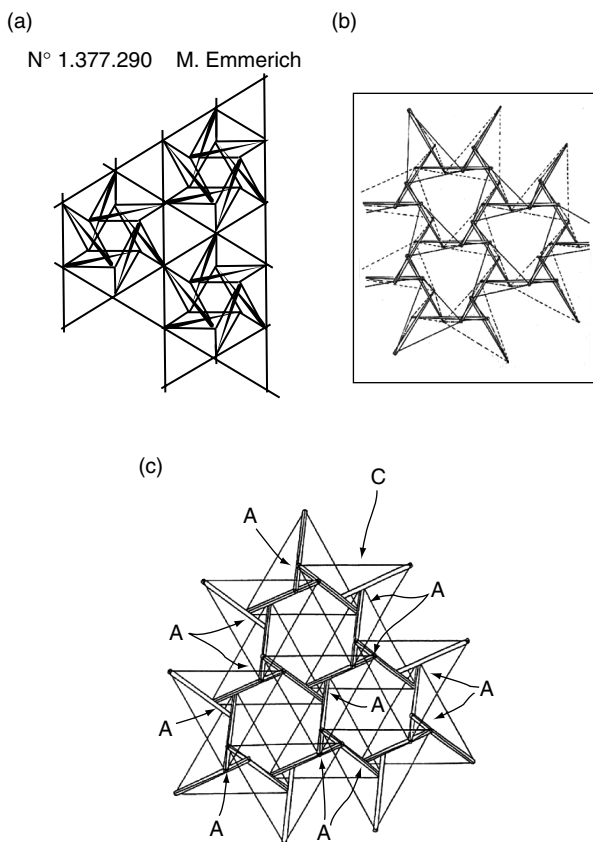


Figure 1. Double-layer tensegrity grids based on tensegrity tripods by (a) Emmerich; (b) Snelson; (c) Kono et al.

Working on the same line, the Mechanics and Civil Engineering Laboratory (LMGC) of Montpellier University, led by Motro, has been hosting, directing and supervising since 1997 some other students working on the same field of planar tensegrity grids, focused on different aspects, like form-finding methods, self-stress states, deployable configurations, construction and active control techniques, optimal dimensioning, etc. In any case, all of them were mainly applied to just two classes of DLTGs: one built with a 2 way grid structure (Fig 2.b), similar to one of Snelson’s planar pices from the 60s (Fig 2.a) and another one made of auto-stable half-cuboctahedron modules (Fig 2c).

Among all those researches, one of the most relevant essays about DLTGs was written by

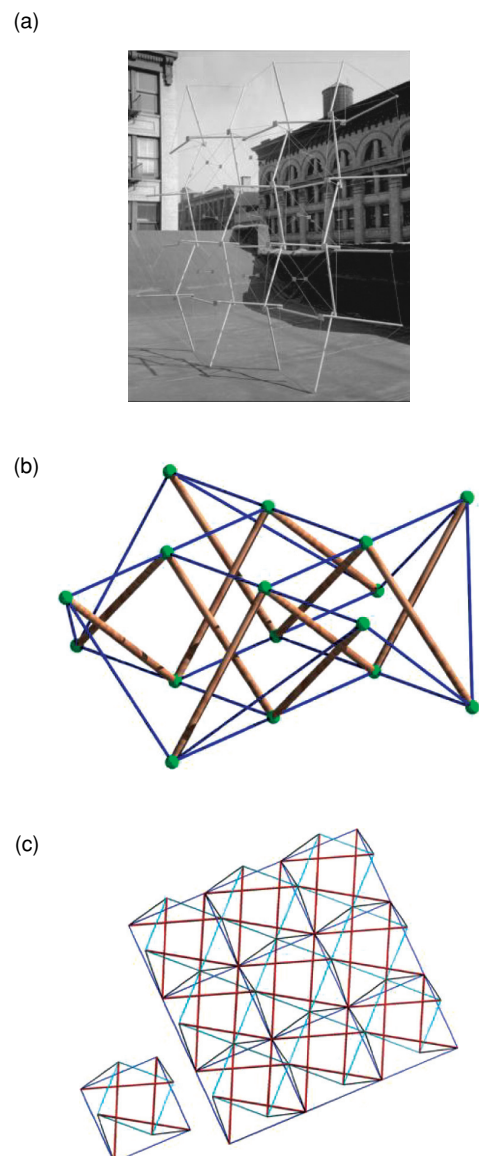


Figure 2. (a) Snelson’s 2-way planar piece (1960). (b) 2-way DLTG. (c) Half-cuboctahedron DLTG.

Raducanu [11] as a part of his PhD thesis in 2001 and the main subject of the Tensarch project. His methodology, original and systematic, was based on the use of inter-depending expanders rather than auto-stable modules, applying topological and geometrical relationships between them. This innovative technique led to a break-through, obtaining new forms never found before, materialized on 15 new grids and a new line of research for future studies.

2. METHODOLOGIES FOR DESIGNING DLTGs

Among all the experiences and studies enumerated on precedent section, two different methods for solving the configuration of DLTGs have generally been applied, and will be summarize in next lines:

2.1. Composition

(*Creation of grids by means of attaching different tensegrity modules one to each other*). These basic cells have been mainly n -fold rotational symmetry prisms and truncated pyramids, constituted by n bars (usually 3 or 4) around a vertical axis. As already exposed, Emmerich [7] proposed many other types that have not been considered thoroughly during the last years. Depending on the type of connection between the modules, they can be classified as follows:

2.1.1. Non-Contiguous struts

Every compressed member is isolated from each other, being connected just by means of members in tension. There are different possibilities:

2.1.1.1. Vertex-to-edge connection

2.1.1.1.1. Unilaterally: (Fig 3.a) Two vertices of both layers (base and top) of a module contact two edges of another one (base and top). Type Ia [4] or A [12].

2.1.1.1.2. Bilaterally: (Fig 3.b) A module contacts with a vertex the edge of another module on one layer (e.g. top) while at the same time is contacted on its edge of the other layer (e.g. bottom) with the vertex of the other module. Type Ib [4] or B [12].

2.1.1.2. Edge-to-edge connection:

(Fig 3.c) Adjacent modules share a portion of their edges on both layers (top and bottom). Type II [4].

2.1.2. Contiguous struts:

(Fig 3.d) Compressed members of one module touch the extremity of other struts of adjacent modules. It could be said that they cannot be classified as pure tensegrity systems due to compressed elements are not discontinuous; nevertheless, Motro [13] claims that they could be considered as a continuum of cables

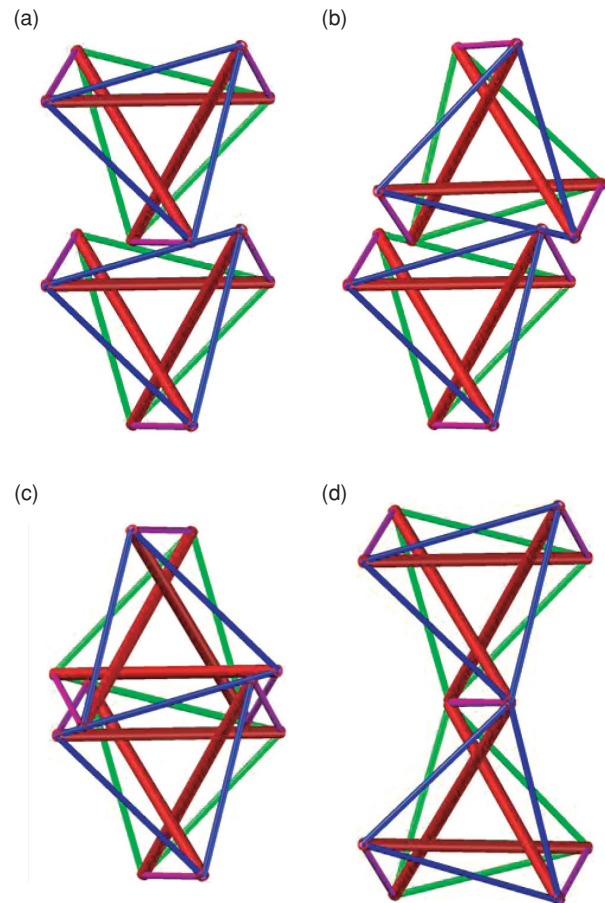


Figure 3. Composition by juxtaposition of tensegrity Simplex: (a) Non-Contiguous V-to-E Unilateral. (b) Non-Contiguous V-to-E Bilateral. (c) Non-Contiguous E-to-E. (d) Contiguous.

comprising some compressed components not touching each other, being each component achieved with a set of several bars.

They are class k tensegrity structures if at most “ k ” compressive members are connected to any node [14]. For example, a non-contiguous strut DLTG is a class $k = 1$ structure because only one compression member makes a node. Analogously, structures with contiguous struts would be $k > 1$.

Attending some other parameters, DLTGs could also be categorized as follows:

- Flexible / Rigid, depending on the number of mechanisms of the modules [8].
- Planar / Domical, depending on the curvature of the grid [5].
- Central / Oblique, depending on the angle between axes and bases of the modules.

2.2. Decomposition

(*Creation of grids without self-stable subsystems joined together, but by means of the integration of bracing members (expanders), composed by*

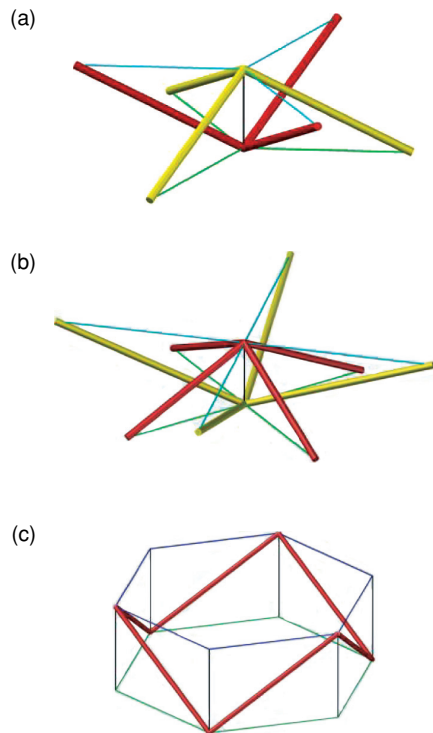


Figure 4. (a) V33 expander. (b) V44 expander. (c) Z6 expander, after Raducanu [11].

compressed struts and tensioned cables, between the top and bottom chords of the grid, obtains a whole structure in equilibrium).

This original approach by Raducanu [11], proposed the use of three different types of expanders depending on their shape: V, Y and Z. The first group was usually denominated Vmn, being m the number of bars meeting at the lower node and n the same at the upper node. For instance, the expander used in the grid of Fig 2.b is a V22. The Y-expanders don't really generate double-layer structures, but triple-layer grids, because they include additional nodes between the top and bottom layers. Finally, the Z group, or Zn expander, is formed by closed chains of n contiguous struts going zigzag between the upper and lower layer of the DLTG. See Fig 4 for some more examples.

3. NEW APPROACH TO DLTGs FROM DLGs

In this paper, a new approach is presented, mainly in geometrical terms because it gives information about the general procedure before contrasting the final geometrics with the self-stress states of the proper form-finding.

Conventional DLGs are usually a composition of regular tessellations (triangles, squares and hexagons filling the space) for either the top, bottom or bracing

members. The composition and representation of DLGs comes from the integration of the three of them; however, Otero [15], [16] proposed their geometrical definition by means of just the mosaic of the bracing members, along with two other factors: the location on the mosaic of the nodes on the bottom and top chords, and the rules of relation for joining them. As a result, new DLGs were defined from different bracing members' tessellations: not only from regular tilings (made with a single type of regular polygon and congruent vertices), but also from semiregular (different types of regular polygons and congruent vertices), demiregular (different types of regular polygons and non-congruent vertices), equifacial (dual of semiregular) and semiequifacial (dual of demiregular) mosaics.

Another research, by Gómez-Jáuregui et al. [17], has been carried out on the countless possibilities and collateral investigations related to that methodology. Although it is not the main aim of the current paper, we will refer to it when defining the nomenclature of the DLGs.

Let's take, for instance, the tessellation 4,6,12 of Fig 5.a, composed by squares, hexagons and dodecagons. This tiling will be considered as the intermediate layer, so the different lines represent the

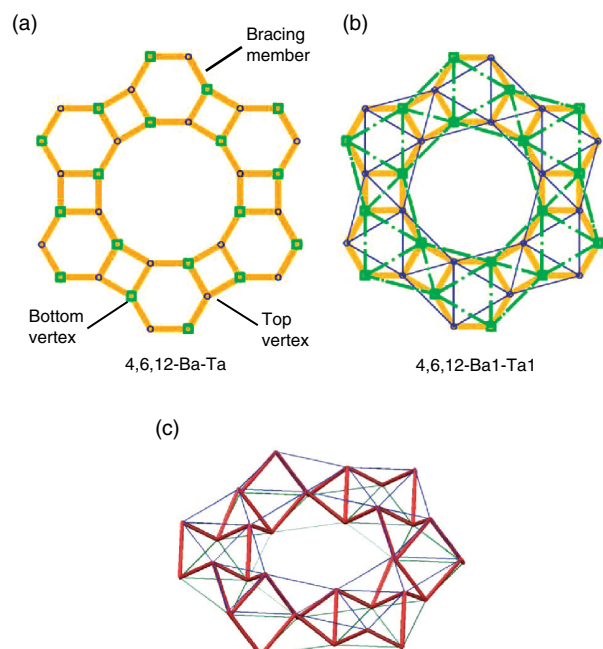


Figure 5. Generation of DLG 4,6,12-Ba1-Ta1 from semi-regular tessellation 4,6,12. (a) Distribution of top and bottom vertices on the mosaic of bracing members. (b) Connection of upper and lower nodes creating top and bottom layers respectively. (c) Perspective of the final configuration of the DLG.

bracing members of the DLG. Then, let's consider that the location of the nodes in both layers (denoted with B for Bottom layer and T for Top layer in 4,6,12-B-T) is alternate (denoted with an a in 4,6,12-Ba-Ta), so the vertices belong or not, alternately, to that particular layer, as it is represented on Fig 5.a (Top vertices as dark circles, Bottom vertices as light squares). Note that this option is possible because every polygon has an even number of sides. Consequently, another rule needs to be followed to join the vertices of each chord; in this case, it will be the easiest and most evident, i.e. connecting to the closest nodes of each polygon (denoted with the number l after the a in 4,6,12-Ba1-Ta1), obtaining the Fig 5.b. The final result is shown in of Fig 5.c in a perspective view.

Henceforth, different methodologies of geometrical configurations can be undertaken: Rot-Umbela manipulation, composition from Emmerich modules, intuitive configuration, truncation and decomposition of nodes, selective consideration of diagonal and vertical members, etc. Special attention is paid to definition of edges and corners, in order to assure the correct stability of the assembly and the transmission of loads to the supporting system.

3.1. Umbela Manipulation

Applied to polyhedra, conventional Umbela manipulation was originally defined as an operation that consists on opening a given direction in the space in such a way that we can obtain a regular polygon with its vertices placed in a plane perpendicular to the chosen direction (Gancedo, [18]). This operation is defined by means of different parameters (Fig 6): a number of new vertices in each direction (p), a semiangle of the regular cone whose vertex is the origin of the tryhedron having a base formed by the new vertices (α) and the orientation of the regular polygons when opening the new vertices ($a_i = b_i = \dots = f_i$). Umbela manipulation permits different polyhedra to be generated from an initial polyhedron, depending on the number of new vertices and the direction of the opening. As is shown in Fig 7, from an octahedron, depending on those parameters, it is possible to obtain a tetrahedron ($p = 2$, $\alpha = 54,736^\circ$, $a_i = b_i = \dots = f_i = 45^\circ$), an hexahedron ($p = 4$, $\alpha = 54,736^\circ$, $a_i = b_i = \dots = f_i = 45^\circ$) or an icosahedron ($p = 2$, $\alpha = 31,717^\circ$, $a_i = b_i = \dots = f_i = 0^\circ$).

3.2. Rot-Umbela Manipulation

In the case of a grid or tessellation, we will define Rot-Umbela manipulation as a particular Umbela manipulation in which the direction of the opening

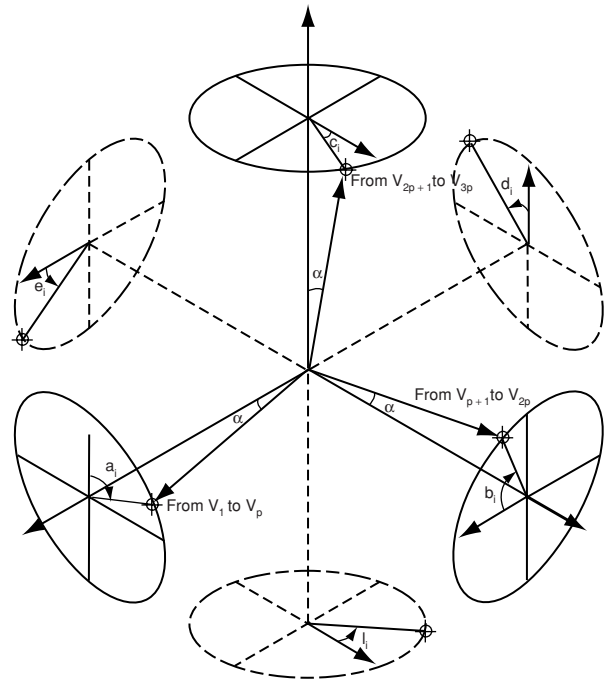


Figure 6. Concept and main parameters of Umbela manipulation (by Gancedo [18]).

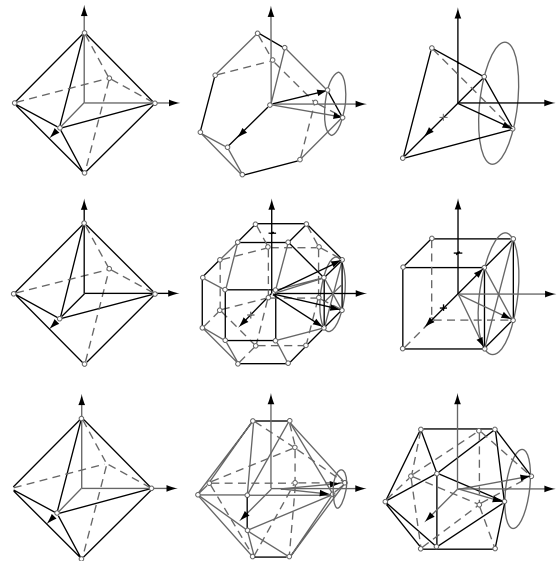


Figure 7. Obtaining a tetrahedron, hexahedron and icosahedron from an octahedron through Umbela manipulation (by Gancedo [18]).

(with a certain amplitude a) is always on the plane of the net and new polygons could also result irregular and rotated (with an angle of rotation r). See Fig 8. Final shape and rotation would be defined by the initial conditions imposed to geometry and pre-stressed state applied to the structure. In some ways, this shape on the vertices of a spatial structure could resemble the 'fans' or 'reciprocal frames' characteristics of the nexorades

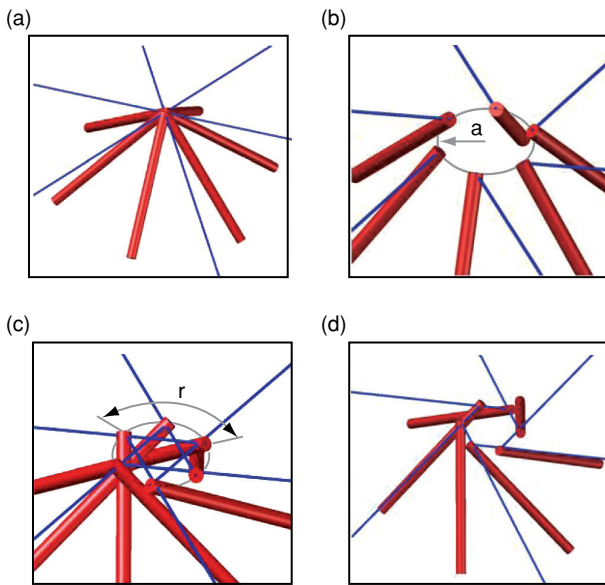


Figure 8. Main parameters and sequence for the Rot-Umbela manipulation. (a) Original configuration of the DLG. (b) Opening (a) of the original upper vertex and generation of six new vertices. (c) Rotation (r) of the new vertices around the original vertex. (d) Reorganization of the top cables to avoid interferences.

or reciprocal structures [19], but obviously without contact between the struts.

For any vertex of valence v , a new polygon of u sides could be generated around it, saying that it has an *umbela valence* u . A vertex has a natural umbela valence if vertex valence (v) and umbela valence (u) coincide ($u = v$). This is the case in Fig 8 ($u = v = 6$) and vertex A and B of semi-regular tessellation 3,4,6 in Fig 9 ($u = v = 4$). An example of the opposite case is vertex C of the same figure ($v = 4, u = 3$).

When talking about DLGs, Rot-Umbela manipulations can be applied to just one of the two layers or both, as well as to all the vertices of the grid or just some ones. Results are countless depending on the complexity of the DLG, and variety of new DLTGs is also remarkable.

For another example, from the DLG generated in Fig 5, let's apply a Rot-Umbela manipulation with natural umbela valence ($u = v = 3$) and initial rotation $r = 150^\circ$. The sequence is exposed in Fig 10, where one of the tripods has been remarked for clearer visualization. Fig 10.a is the original configuration of

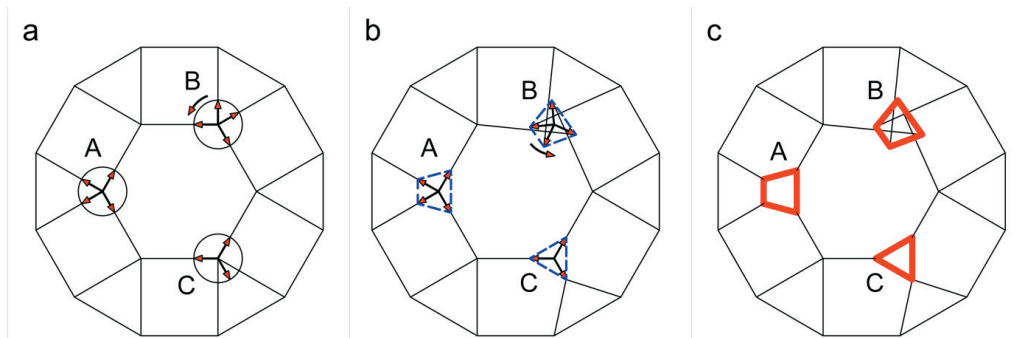


Figure 9. Rot-Umbela manipulation on semi-regular tessellation (3,4,6) with natural umbela valence ($u = v = 4$) in vertices A and B, but different umbela valence ($u = 3, v = 4$) in vertex C.

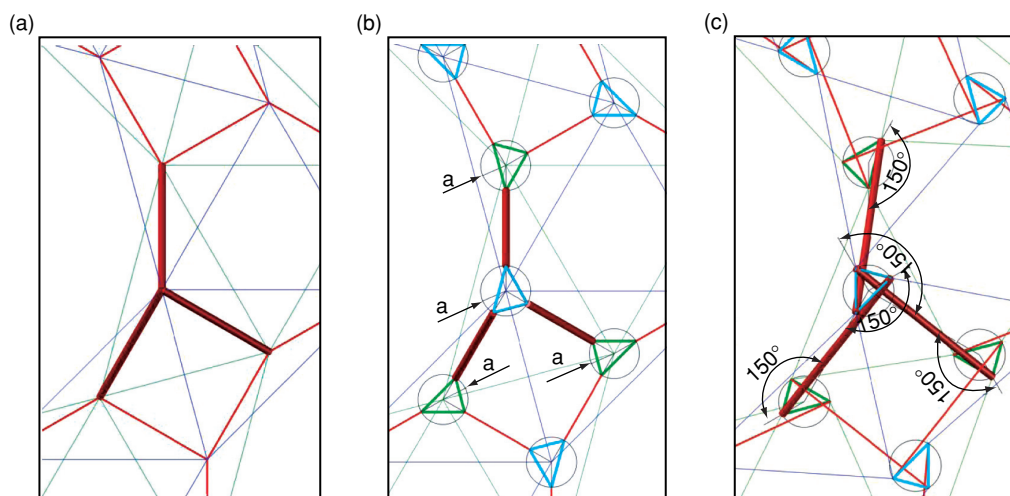


Figure 10. Detail in a sequence of Rot-Umbela on vertices of DLG 4,6,12-Ba1-Ta1. (a) Original DLG. (b) Opening of the vertex and creation of new vertices (c) Rotation of the new vertices around the original node.

the DLG. In Fig 10.b the opening (a) is executed in every vertex of the grid with natural umbela valence. In this case, a is 20% of the length of the horizontal projection of the struts. Finally, Fig 10.c shows the configuration of the DLTG after the application of the rotation ($r = 150^\circ$)

As can be observed in Fig 11.a, this manipulation generates irregular polygons (triangles with three different angles). If applied only on the bottom layer, the tensegrity grid of Fig 11.b would be obtained. Note that compressed elements are sets of tripods (class $k = 3$) not touching each other.

If a similar Rot-Umbela manipulation is applied also to top layer, a different DLTG would be attained

(Fig 11.c), a non-contiguous configuration (class $k = 1$), composed of T-tripods similar to those used in grids of Fig 1.

It is remarkable that some configurations of DLTGs already known respond to the final result of a Rot-Umbela manipulation. It is possible to apply it to DLG of Le Ricolais, easily obtained after Otero's rules from the regular mosaic 6^3 , as illustrated in Fig 12.a and b, which nomenclature would be 6^3 -Ba1-Ta1. From this grid, the hexagonal DLTG made of tripods of Fig 1 can be generated applying the following parameters:

- Natural umbela valence: $u = v = n = 3$ on upper and bottom layer, n being the number of struts concurring at each vertex (Fig 12.c).

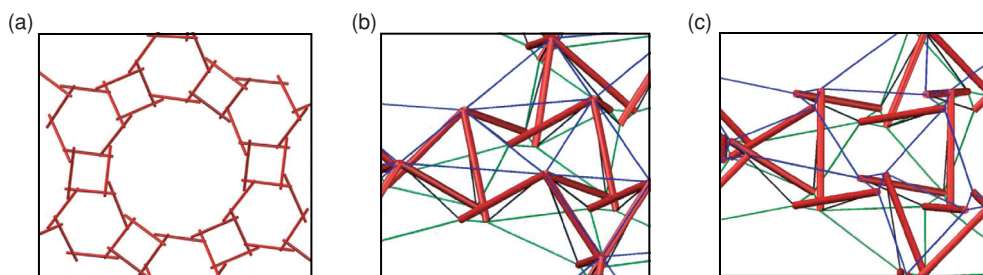


Figure 11. DLTG from 4,6,12-Ba1-Ta1: (a) Rot-Umbela manipulation ($u = v = 3$), plan view (showing just the struts). (b) Rot-Umbela only on the lower layer, perspective. (c) Perspective of a Rot-Umbela on both layers.

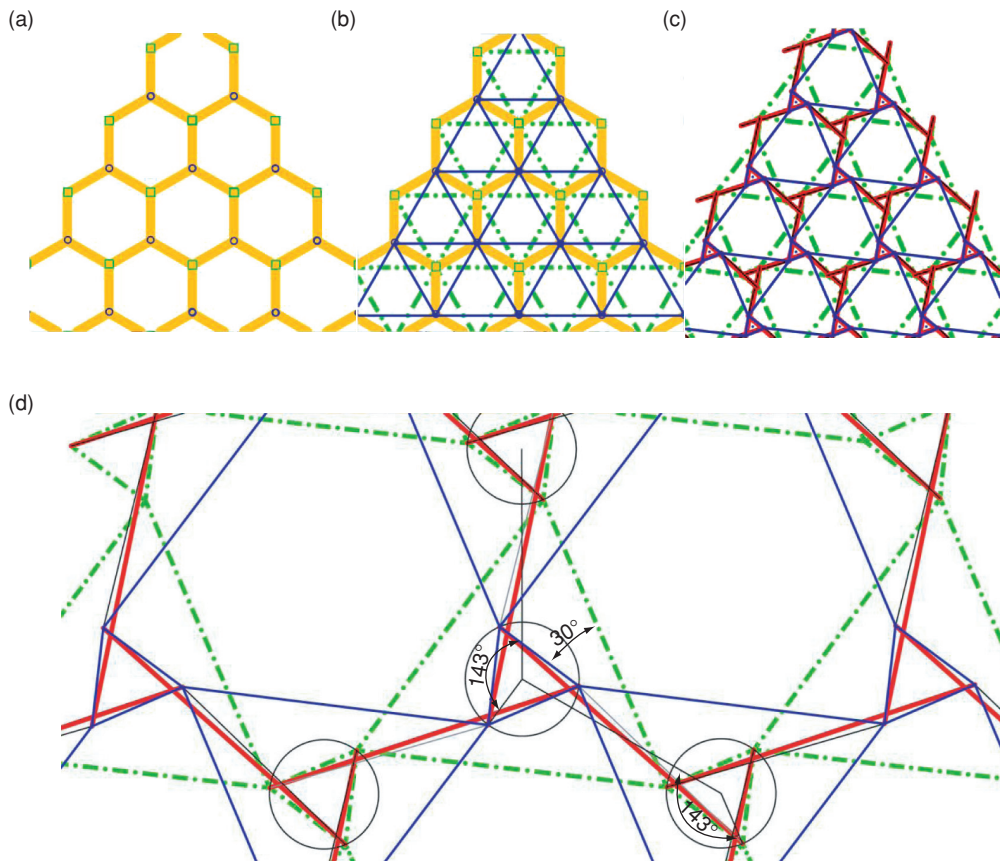


Figure 12. Obtaining Kono et al.'s DLTG from DLG 6^3 -Ba1-Ta1 (Le Ricolais) with Rot-Umbela manipulation.

- Amplitude (a) of the opening is not crucial in this case and can be optional.
- Rotation: $r = 143^\circ$ (Fig 12.d).

Another example corresponds to Fig 13, which is the application of Rot-Umbela manipulations on DLG $(4^4)^{45}$ -Ba1-Ta1 (Space Deck or square-on-square grid) of Fig 13.a-b, with the following parameters:

- Natural umbela valence: $u = v = n = 4$ on upper layer, n being the number of struts concurring at each vertex.
- Amplitude of the opening: $a = L \cdot \cos(\pi / n) = L \cdot \sqrt{2} / 2$, L being the length of the horizontal projection of the struts (Fig 13.c).
- Rotation: $r = 90^\circ + 180^\circ / n = 135^\circ$ (Fig 13.d-e).

As can be observed in Fig 13.a and b, originally four struts are meeting at each upper vertex. In Fig 13.c, the opening of those vertices is applied with valence $v = 4$ so, for instance, vertex 1 becomes

vertices 1a, 1b, 1c and 1d. The next two images show the rotation of these new nodes in two phases: Fig 13.d and e present a rotation of 90° and 135° respectively. Final configuration (Fig 13.f) corresponds to the DLTGs composed of half-cuboctahedrons already shown in Fig 2.c.

As already mentioned, final geometry of the DLGs is usually achieved by means of a rotation of the polygons opened around the vertices. It is widely known that in tensegrity prisms or pyramids, there is always a twist angle α between both bases, depending on the number of sides or struts of the system (n), following the well-known formula $\alpha = 90^\circ - 180^\circ / n$. In Rot-Umbela manipulations there is a twist angle as well. In any case, as in tensegrity structures, it is possible to correct that rotation by means of changing the configuration and/or number of the tendons of the structure or its initial state of self-stress.

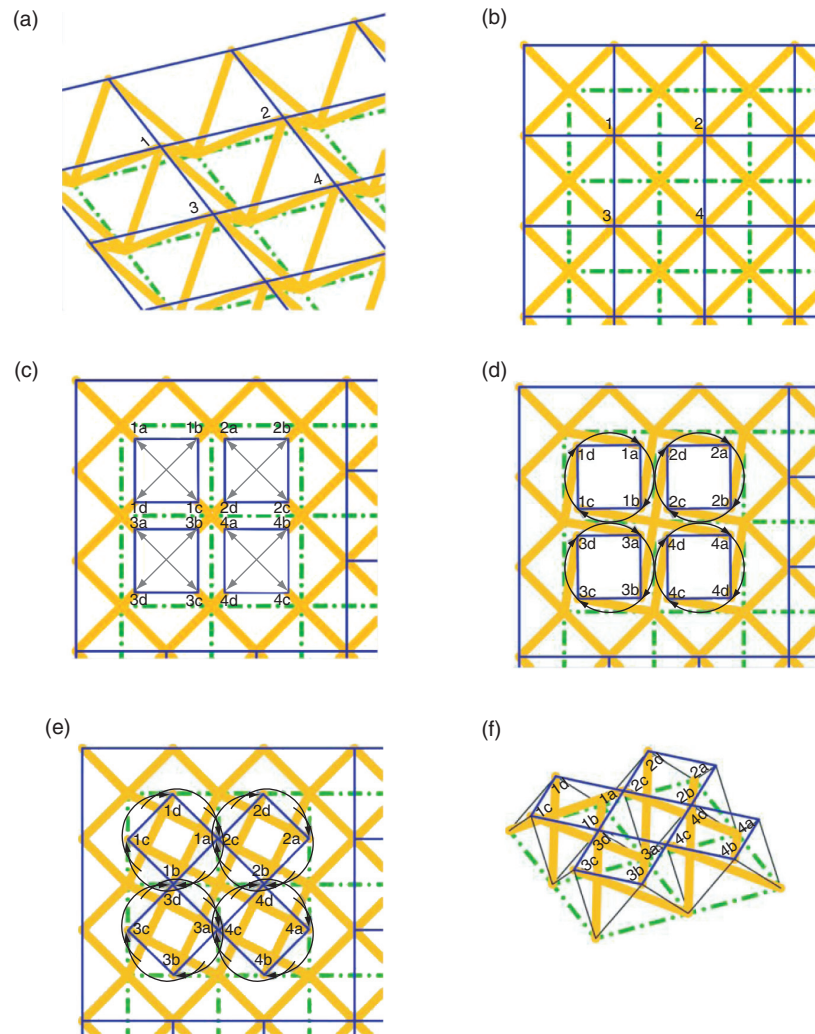


Figure 13. Obtaining DLTG of half-cuboctahedrons from DLG $(4^4)^{45}$ -Ba1-Ta1 (Space Deck or s-on-s grid) with Rot-Umbela manipulation.

3.3. Analysis of the grids

In order to achieve the correct and stable configuration of the DLTGs proposed in precedent sections, it is essential to prove its stability and equilibrium by means of a study of the structure. Special attention has to be paid to edges of the grid and boundary conditions, because their configuration is critical for providing the stability and equilibrium to the whole system.

Analysis of the structures has been done with three different approaches in order to compare results: firstly, a numerical method for calculating the number of mechanisms and states of self-stress by consecutively solving two homogenous linear systems, strongly inspired on the studies on the subject [20] and the numerical methods developed by Tran and Lee [21]; then, a real time implementation of a discrete element method with mass-spring systems [22]; and finally, a modified dynamic relaxation algorithm applied to clustered tensegrity structures [23].

Structural behavior of the grids used as examples in this paper will be exposed in further communications (e.g. deflections, reactions, response to external loads, etc.). However, it is noteworthy the fact that differences between the grid by Kono et al. (Fig 14.b)

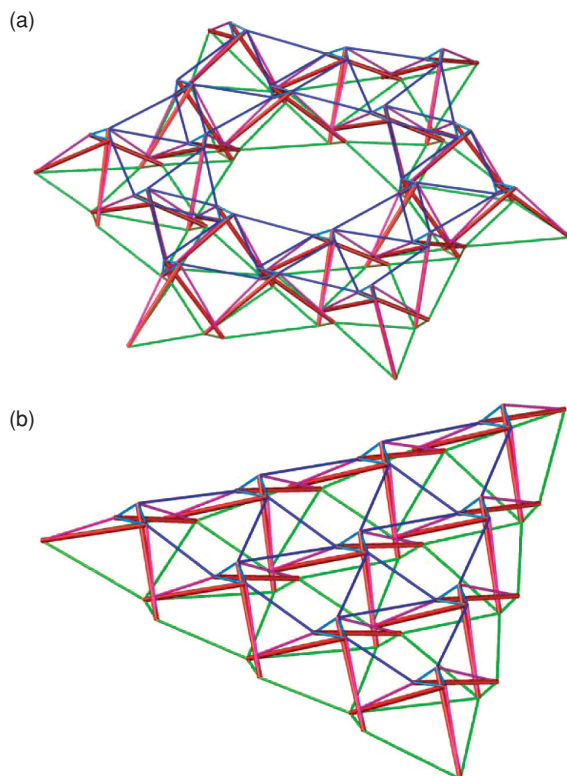


Figure 14. (a) Rot-Umbela manipulation on 4,6,12-Ba1-Ta1.
(b) Rot-Umbela manipulation on 6³-Ba1-Ta1.

and DLTG obtained by Rot-Umbela manipulation on DLG 4,6,12-Ba1-Ta1 (Fig 14.a) are not significant in terms of internal mechanisms and states of self-stress. While the first grid has $m = 18$ infinitesimal internal mechanisms with 10 tensegrity tripods, the second one has almost the same number of mechanisms ($m = 19$) with nearly the double of tensegrity tripods (18). Analysis of unilateral mechanisms was done according to Maurin et al. [92], but not one was found in any of the grids. In addition, both of them have only one feasible state of self-stress.

DLTG obtained from Rot-Umbela manipulation on 4,6,12-Ba1-Ta1 leads to a structure composed by 108 nodes (54 in each layer) and 300 elements (54 bracing struts, 96 lower cables, 96 upper cables and 42 bracing wires), as can be observed in Fig 15. Moreover, some distinctions have been made between some elements of each layer: triangular wires respond to the short tendons that form small triangles in each layer; inner upper wires are those of the internal dodecagon of the top layer; outer upper wires are those of the external dodecagon of the top layer.

From this distribution and with the geometry shown in Fig 15, a form-finding analysis was carried out in order to obtain a feasible state of self-stress and the final location of the nodes. Using the three calculation methodologies exposed above, it was possible to solve this questions. Firstly, it was obtained the Equilibrium Matrix and, after applying an iterative method, the kernel or null space of the matrix was obtained. Being a one-dimensional space, that was the only feasible state of self-stress. Then, with the dynamic relaxation algorithm, modified coordinates (with very small variations compared to the original position of the nodes) and internal forces (shown in Fig 16) were obtained.

It can be observed that the solution is compatible with the rigidity of struts, i.e. in compression (first group of elements with negative values) and with the cables, i.e. only tension (rest of the members, all with non-negative values). Thus, the mechanical validity of the obtained configuration is proved.

In the particular case of this flat DLTG, values of some tensions at inner and outer members (elements 151 to 191 of Fig 16) are very low compared to tensions in other elements of the grid. However, they are important in order to minimize the number of internal mechanisms. For example, without those 12 extra inner and outer upper wires, the internal mechanisms (m) would be 30 instead of 18. Besides, there could be some other occasions (domical

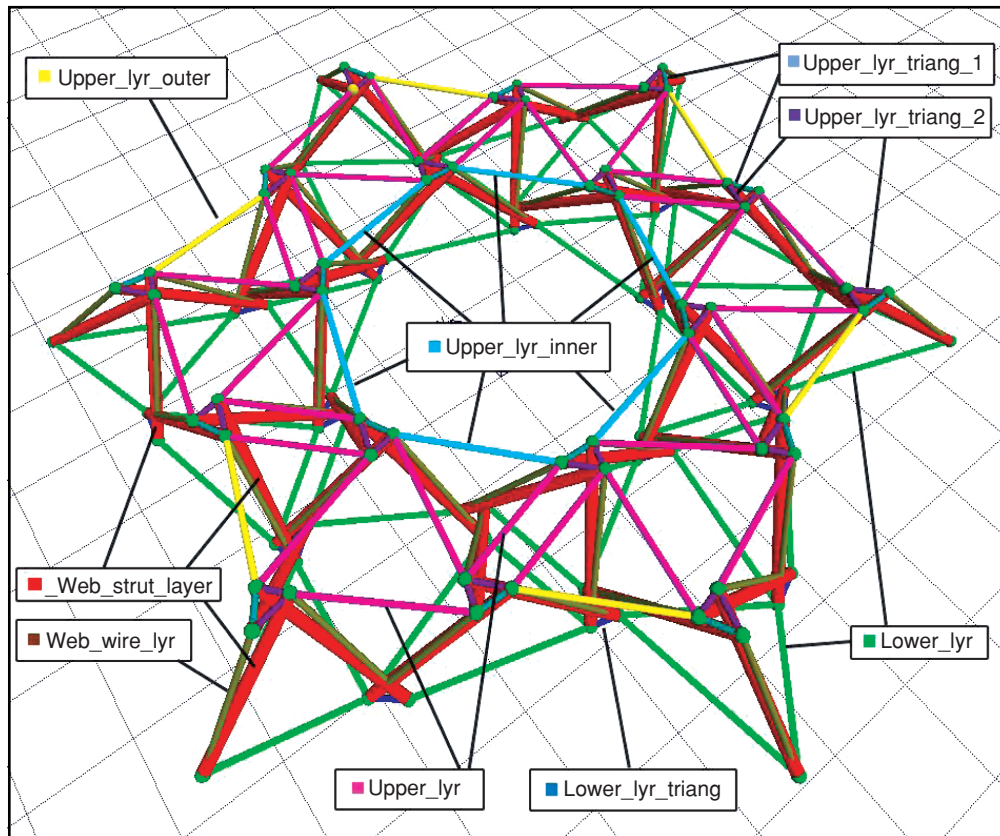


Figure 15. DLTG obtained from Rot-Umbela manipulation on 4,6,12-Ba1-Ta1.

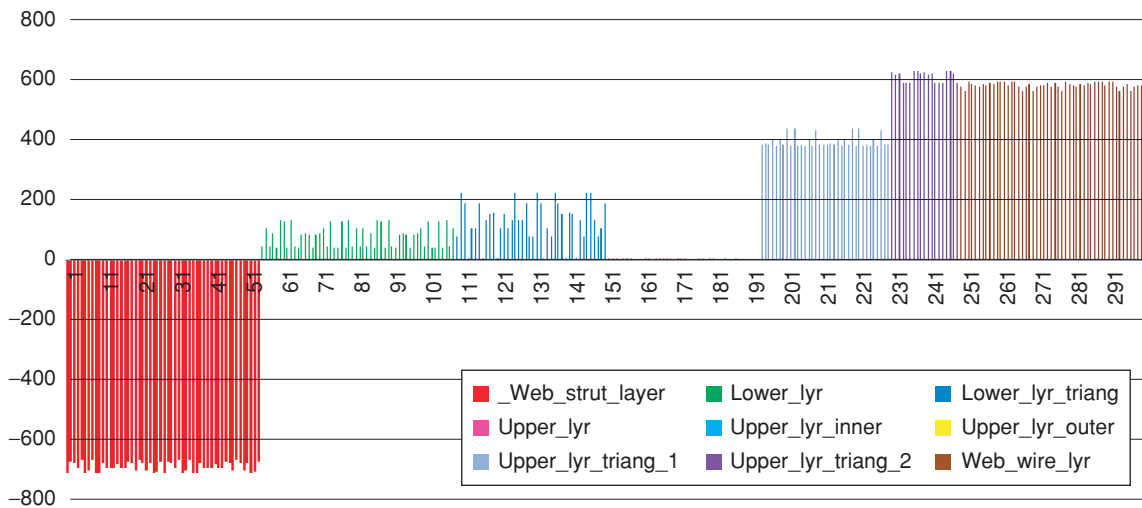


Figure 16. Internal forces of the different elements of the DLTG obtained from Rot-Umbela on 4,6,12-Ba1-Ta1.

configurations or external loading cases) where those elements could have higher values. Nonetheless, this situation of having very small tensions on some members of the grid is common to some other DLTGs. For example, the one by Kono et al. (Fig 14.b), whose internal forces are presented in Fig 17, where elements 101 to 116 and 121 to 126 are barely in tension in comparison with the other cables of the grid.

The other case exposed on this paper, DLTG composed of half-cuboctahedrons (Fig 13.d), is very different due to the fact that it is not class $k = 1$ and, thus, as bars are in contact, it is much more rigid. Some other authors have already studied this grid in depth [5], [21], [24], whose results coincide with the one obtained here (several states of self-stress and just one internal mechanism $m = 1$).

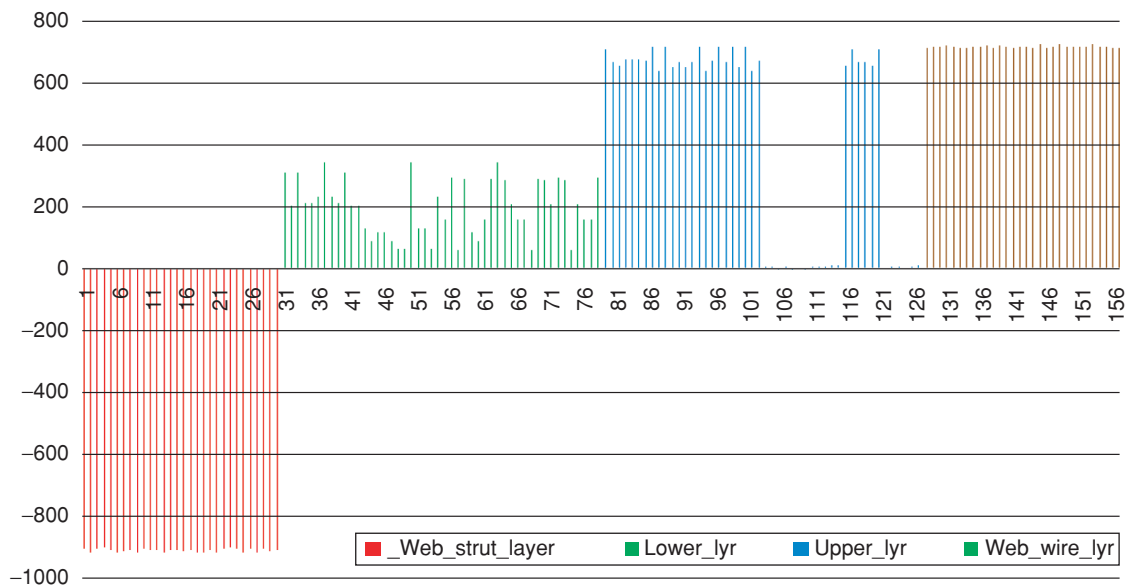


Figure 17. Internal forces of the different elements of the DLTG by Kono et al.

4. FUTURE RESEARCH

Analytical comparison between conventional and tensegrity double-layer grids are also being established depending on different factors: weight, resistance, deformation, clearance, economy of materials, etc. Obviously, important advantages and disadvantages are argued depending on the application of the final mesh. Thus, some conclusions about the possible application of the new DLTGs are being arisen and will also be presented in the future.

5. CONCLUSIONS

Even though for the last years there have been several proposals for designing DLTGs, most of them have been obtained with a methodology based on composition, attaching tensegrity modules one to each other. As shown by Raducanu, there are other possibilities with interesting geometry and applications. A new approach is shown in this work, parting from tessellations that originate conventional DLGs, which lower and/or upper layers are modified with Rot-Umbela manipulations. From this point, a new catalogue of possibilities is open for creating new arrangements of DLTGs.

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