New configurations for double-layer tensegrity grids

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Summary

Double-layer tensegrity grids (DLTGs) may be defined as tensegrity spatial systems containing two parallel networks of members in tension forming the top and bottom chords, whose nodes are linked by vertical and/or inclined web members in compression (although some of them could also be in tension).

As an introduction, it is presented a brief perspective of the historical proposals for DLTGs over the last 50 years, including an extended bibliography with the most important contributions of different authors concerning tensegrity planar pieces.

In this paper, a new approach is described, mainly in geometrical terms, taking conventional double-layer grids (DLGs) as the starting point of the research. They are composed of three layers: top, bottom and inner chords (usually regular tessellations); however, Otero proposed a new methodology for their composition from the mosaic of the diagonal web and additional laws. Following this scheme, this paper shows new rules for generating original DLG defined not only from regular, but also from semiregular, demiregular, equifacial (dual of semiregular) and semiequifacial (dual of demiregular) mosaics. From them, a new technique, known as Rot-umbela Manipulation, is applied to any of those DLGs to obtain their tensegrity form, opening and endless catalogue of DLTGs.

Keywords

tensegrity, structures, double-layer, grids, design, tessellations, rot-umbela manipulation

Theme

Structural & Architectural Design – Action engineering / conventional loads – Device / bearing

1. Introduction

Tensegrity is a principle based on self-stressed and auto-stable structures composed by isolated components in compression inside a net of continuous tension, in such a way that the compressed members (usually bars or struts) do not touch each other and the pre-stressed tensioned members (usually wires or even tensile membranes) delineate the system spatially *[1]*. Based on this concept, double-layer tensegrity grids (DLTGs) are defined as tensegrity spatial systems containing two parallel networks of members in tension (top and bottom chords), whose nodes are linked by vertical and/or inclined web members in compression and tension.

These kinds of structures are being taken into account in the last years with increasing frequency for the construction of several roofs, covers and even bridges (like the Kurilpa Bridge in Brisbrane, Australia). Even though some of them could not be considered as pure tensegrity structures, there is a rising sensibility to their application with engineering and architectural purposes.

In this work, the origins, evolution and new trends on DLTGs over the last few years will be presented. It will be back up whit continuous references to an extended bibliography included at the end of the paper, gathering the most important contributions of different authors concerning tensegrity planar pieces. Then, the basic two methodologies used at the moment for the generation of DLTGs will be exposed, based on composition and decomposition techniques. After that, a new approach will be proposed, parting from conventional DLGs and applying to them a new kind of operation, denominated rot-umbela manipulation. It will be explained that some of the current tensegrity grids could be obtained by rot-umbela manipulations. Finally, some notes about other future proposals, analysis and conclusions are presented as part of a research with larger implications.

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2. Historical Background

Since the controversial discovery of Tensegrity *[2]* in the 1940s-50s, many configurations of tensegrity grids have been proposed. In the early years, mainly Fuller *[3]*, Snelson *[4]* and Emmerich *[5]*,*[6]* started to design and create models of several tensegrity planar pieces, utterly different from the more typical spherical or prismatic tensegrity systems. Some of them *[7]*,*[8]* were not shown publicly until recent years, even though they could be of interest for study and analysis. In the 70s, mainly Pugh *[9]* and Vilnay *[10]* proposed some other tensegrity nets, although it was not until the next decade when Hanaor *[11-17]* and Motro *[18-23]* took a more structural and mechanical approach and studied the form-finding, resistance and stability of the double-layer tensegrity grids (DLTG), where the bars are confined between two parallel layers of cables. The former experienced basically with the juxtaposition of tensegrity prisms (for planar pieces) and truncated pyramids (for domical configurations), avoiding contacts between struts like Emmerich did; meanwhile the latter studied the same tensegrity pyramids (mainly the same half cuboctahedrons showed by Emmerich *[5]* in his first patent) but for planar grids, by means of joining the ends of some struts. Besides, Nestorovic *[24]*,*[25]* also developed some proposals for integrally tensioned domes made of tensile double-flanged network.

Analogously, Emmerich *[26-28]* published a complete and resourceful compendium of the theories, structures, models and projects carried out by himself and his students. One of the sections of that book was dedicated exclusively to "self-stressed planar nets", although in a very restrictive manner, taking into consideration just some of the possible kinds of DLTGs. There, he enumerated many configurations based on prismatic, anti-prismatic, anti-pyramidal, interlaced and inter-penetrated tensegrity modules creating several and variable tessellations.

Following their steps, at the end of the last century, some other studies were undertaken; although not very different to the pre-existing configurations, they have made an important contribution to the different manners of analyzing these kinds of structures. One of the most prolific authors, Wang B.B. *[29-37]* has been since 1996 analyzing thoroughly the combinations of modules to generate several types of DLTGs, depending on the kind of simplex to use, the connection between them, the contiguity of the struts, the rigidity of the systems, the location of the supports, etc. In other words, he continued Emmerich's task from a more structural rather than architectural approach.

Hilyard and Lalvani *[38]* called attention to some basic grids based on the filling of several tessellations with space cells, which they called Emmerich-type structures, constructed from prismatic tensegrity modules.

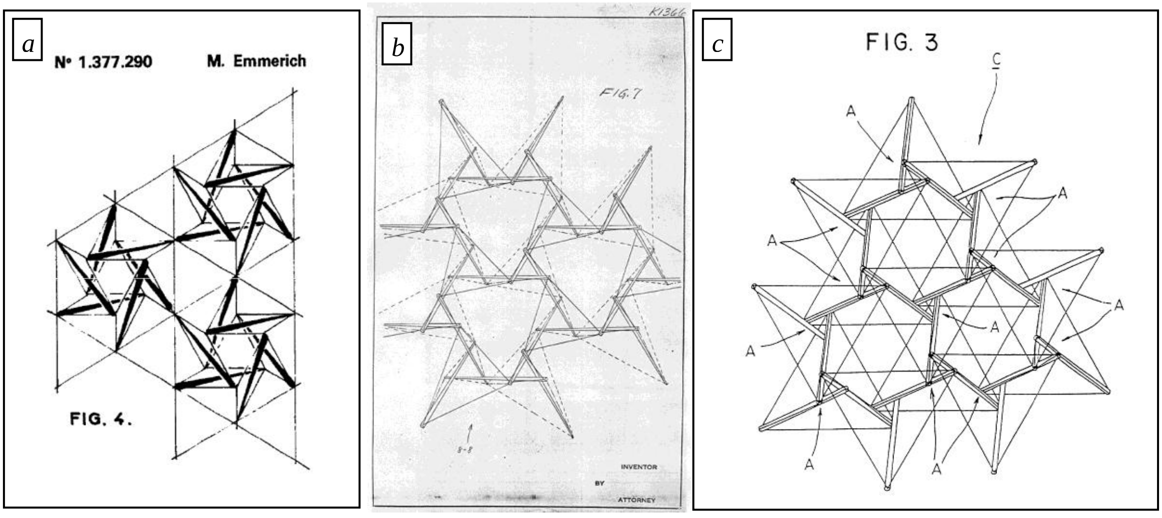

Figure 1. Double-layer tensegrity grids by Emmerich (a), Snelson (b) and Kono&Kunieda (c) based on tensegrity tripods.

At the same time, Kono and Kunieda *[39-44]* experimented with a single kind of tensegrity grid, based on the use of tripods or truncated pyramids of 3 bars each, somehow similar to design #4 of Emmerich's patent *[5]* (although with some vertex-to-vertex connections) (Figure 1.a) and Snelson's abandoned patent *[7]* (Figure 1.b). Different geometries were tested, depending on the attachments of the wires of the bigger base (direct to the edges of the struts, to the GC of the base or to some intermediate points of the bisectrices of the base). A big-scale model composed by 33 triangular modules, with an 80m2 covered area, was constructed at the end of the experiment, including for their assembly a newly proposed member joint system. Finally, the study ended with the grant of a patent *[44]* which also included configurations of 4 strut modules and domical representations of that specific DLTG.

Working on the same line, the Mechanics and Civil Engineering Laboratory (LMGC) of Montpellier University, leaded by Motro, has been hosting, directing and supervising since 1997 some other students working on the same field of planar tensegrity grids: Quirant *[45-47]*, Smaili *[48-51]*, Sánchez *[52-54]*, Bouderbala *[55]*, Vassart *[56]*, Averseng *[57-60]*, Djouadi *[61-63]*, Kebiche *[64]*, Le Saux *[65]*, etc. Their essays were focused on different aspects, like form-finding methods, self-stress states, deployable configurations, construction and active control techniques, optimal dimensioning, etc. In any case, all of them were mainly applied to just two classes of DLTGs: one built with a 2 way grid structure (Figure 2.b), similar to one of Snelson's planar pices from the 60s (Figure 2.a) and another one made of auto-stable half-cuboctahedron modules (Figure 2c).

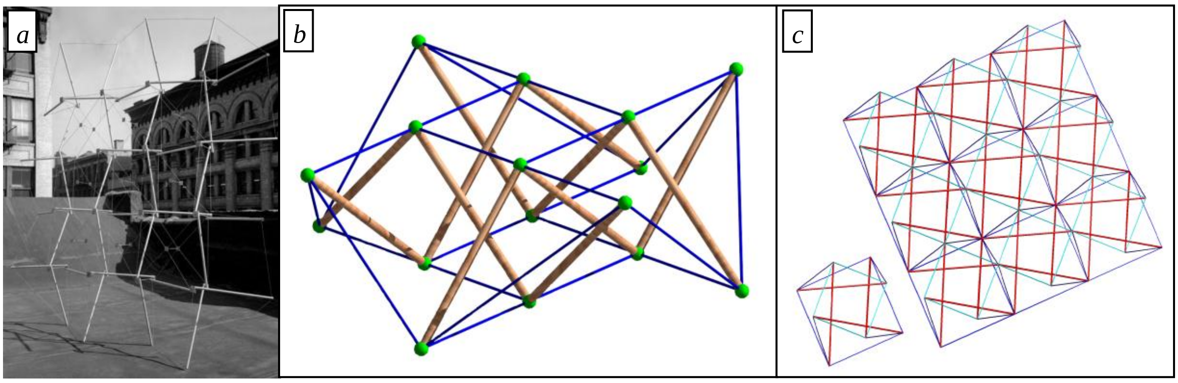

Figure 2. a) Snelson's 2-way planar piece (1960). b) 2-way DLTG. c) Half-cuboctahedron DLTG

Among all those researches, one of the most relevant essays about DLTG was written by Raducanu *[66]* as a part of his PhD thesis in 2001 and the main subject of the Tensarch project. After briefly exposing the main concepts and characteristics of tensegrity systems, and resuming precedent similar works dealing with DLTGs, he explained his own approach to find new configurations. The methodology, original and systematic, was based on the use of inter-depending expanders rather than auto-stable modules, applying topological and geometrical relationships between them. This innovative technique leaded to a break-through, obtaining new forms never found before, materialized on 15 new grids and a new line of research for future studies. This theoretical effort was carried out by different means and integrating powerful tools like Formian, Autocad, Matlab, Abaqus, etc. It was finally applied to the construction of a big-scale prototype, in order to find a feasible way of incorporating tensegrity structures into new architecture. As a result, Raducanu and Motro *[67]* filled that same year an application for a patent exposing the outcomes of the research.

There have been some other interesting proposals, like Passera and Pedretti's *[68]* project for Swiss Expo 2001 or grid shells designed by Addriansens and Barnes *[69]*; however they will not been taken into account for not being proper DLTGs but compositions of octahedral cells like originally exposed Pugh *[9]*.

In any case, it is remarkable that researches on conventional DLTGs have not decreased, and recently some authors have been undertaking studies on this kind of structures. It is the case of Panigrahi, Gupta and Bhalla *[70], [71]*, constructing and testing a dismountable tensegrity grid for possible deployment as light-weight roof structures; or Tran and Lee *[72-74]*, dealing with the form-finding and initial self-stress of tensegrity grids (essentially the same ones studied in the LMGC of Montpellier).

3. Methodologies for designing DLTG

Among all the experiences and studies enumerated on precedent section, two different methods for solving the configuration of DLTG have generally been applied, and will be summarize in next lines:

3.1. Composition

(*Creation of grids by means of attaching different tensegrity modules one to each other*). These basic cells have been mainly n-fold rotational symmetry prisms and truncated pyramids, constituted by n bars (usually 3 or 4) around a vertical axe. As already exposed, Emmerich *[26]* proposed many other types that have not been considered thoroughly during the last years. Depending on the type of connection between the modules, they can be classified as follows:

3.1.1. Non-Contiguous struts: Every compressed member is isolated from each other, being connected just by means members in tension. There are different possibilities:

3.1.1.1. Vertex-to-edge connection:

3.1.1.1.1. **Unilaterally**: two vertices of both layers (base and top) of a module contact two edges of another one (base and top). Type Ia or A *[11]*, *[75]*.

3.1.1.1.2. **Bilaterally**: a module contacts with a vertex the edge of another module on one layer (e.g. top) while is contacted on its edge of the other layer (e.g. bottom) with the vertex of the other module. Type Ib or B *[11]*, *[75]*.

3.1.1.2. **Edge-to-edge connection**: adjacent modules share a portion of their edges on both layers (top and bottom). Type II after Hanaor *[11]*.

3.1.2. Contiguous struts: Compressed members of one module touch other struts of adjacent modules. It could be said that they cannot be classified as pure tensegrity systems due to compressed elements are not discontinuous; nevertheless, Motro *[21]* claims that they could be considered as a continuum of cables comprising some compressed components not touching each other, being each component achieved with a set of bars.

They are class k tensegrity structures if at most “k” compressive members are connected to any node *[76]*. For example, a non-contiguous strut DLTG is a class 1 structure because only one compression member makes a node.

Attending some other parameters, DLTGs could also be categorized as follows:

- Flexible / Rigid, depending on the number of mechanisms of the modules *[36]*.
- Planar / Domical, depending on the curvature of the grid *[18]*.
- Central / Oblique, depending on the angle between axes and bases of the modules.

3.2. Decomposition

(*Creation of grids without self-stable subsystems joined together, but by means of the integration of web members (expanders), composed by compressed struts and tensioned cables, between the top and bottom* *chords of the grid, obtaining a whole structure in equilibrium*).

This original approach by Raducanu *[66]*, proposed the use of three different types of expanders depending on their shape: V, Y and Z. The first group was usually denominated Vmn, being *m* the number of bars meeting in the lower node and *n* the same in the upper node. For instance, the expander used in the grid of Figure 2.b is a V22. The Y-expanders don't really generate double-layer structures, but triple-layer grids, because they include additional nodes between the top and bottom layers. Finally, the Z group, or Zn expander, is formed by closed chains of n contiguous struts going zigzag between the upper and lower layer of the DLTG. See Figure 3 for some more examples.

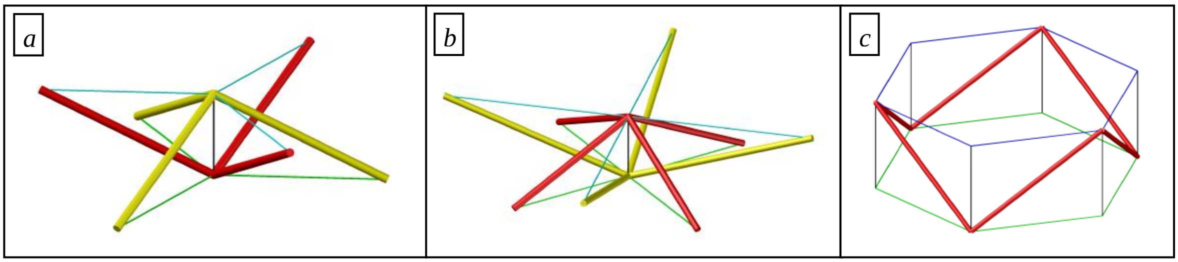

Figure 3. a) V33 expander. b) V44 expander. c) Z6 expander, after Raducanu [66]

4. New approach to DLTG from DLG

In this paper, a new approach is presented, mainly in geometrical terms because it gives information about the general procedure before contrasting the final geometrics with the self-stress states of the proper form-finding.

Conventional double-layer grids (DLG) are usually a composition of regular tessellations (triangles, squares and hexagons filling the space) for either the top, bottom or inner chords. The composition and representation of DLG comes from the integration of the three of them; however, Otero *[77], [78]* proposed their geometrical definition by means of just the mosaic of the diagonal web, along with two other factors: the location on the mosaic of the nodes on the bottom and top chords, and the rules of relation for joining them. As a result, new DLG were defined from different web members' tessellations: not only from regular, but also from semiregular, demiregular, equifacial (dual of semiregular) and semiequifacial (dual of demiregular) mosaics.

Another research on the countless possibilities and collateral investigations related to that methodology is also being carried on, but it is not the main aim of the current paper.

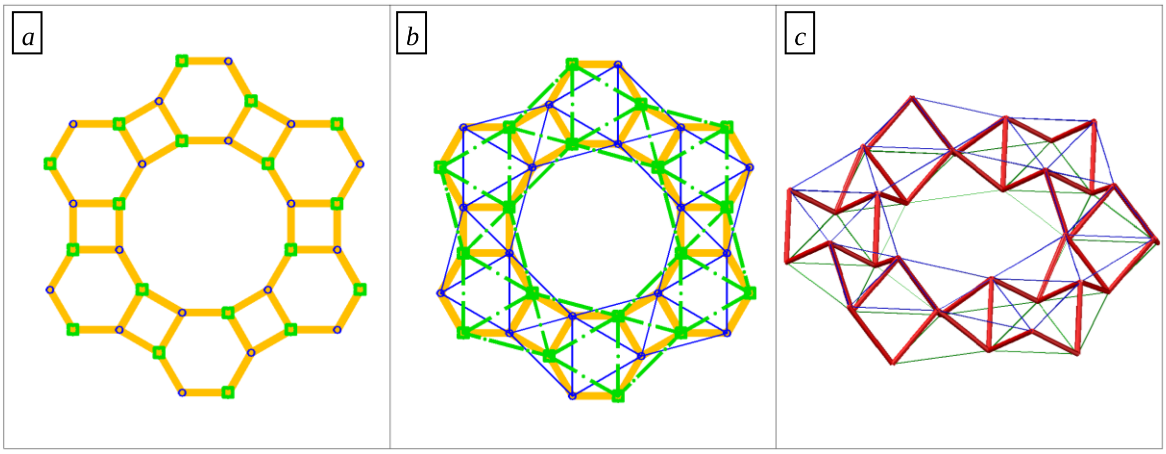

Figure 4. Generation of DLG from semi-regular tessellation 4,6,12.

Let's take, for instance, the tessellation 4,6,12 of Figure 4.a, composed by squares, hexagons and dodecagons

It will be considered as the web layer, so the different lines represent the diagonals of the DLG. Then, let's consider that the location of the nodes is alternate, so if a vertex belongs to the top layer, its neighbors belong to the bottom layer and vice versa, as it is represented on Figure 4.a. Note that this option is possible because every polygon has an even number of sides. Consequently, another rule needs to be followed to join the vertices of each chord; in this case, it will be the easiest and most evident, i.e. connecting to the closest nodes of each polygon, obtaining the Figure 4.b. The final result would be the double-layer grid of Figure 4.c.

Henceforth, different methodologies of geometrical configurations can be undertaken: Rot-umbela Manipulation, composition from Emmerich modules, intuitive configuration, truncation and decomposition of nodes, selective consideration of diagonal and vertical members, etc. Special attention is paid to definition of edges and corners, in order to assure the correct stability of the assembly and the transmission of loads to the supporting system.

4.1. Rot-Umbela Manipulation

Applied to polyhedra, Umbela Manipulation is defined as an operation that consists on opening a given direction in the space in such a way that we can obtain a regular polygon with its vertices placed in a plane perpendicular to the chosen direction (Gancedo, *[79], [80]*). In the case of a grid or tessellation, we will define Rot-Umbela Manipulation as a conventional Umbela Manipulation in which the direction of the opening is always on the plane of the net and new polygons could also result irregular and rotated (final shape and rotation would be defined by the initial conditions imposed to geometry and pre-stressed state applied to the structure). For any vertex of valence *v*, a new polygon of *u* sides could be generated around it, saying that it has an *umbela valence u*. If vertex valence and umbela valence are coincident (u=v), as seen in vertex A and B of demi-regular tessellation of
Figure *5*, it is said that the vertex has a *natural* umbela valence. An example of the opposite case is vertex C of the same figure (v=4, u=3) and another one is illustrated on Figure 6.a, where v=5 and u=3.

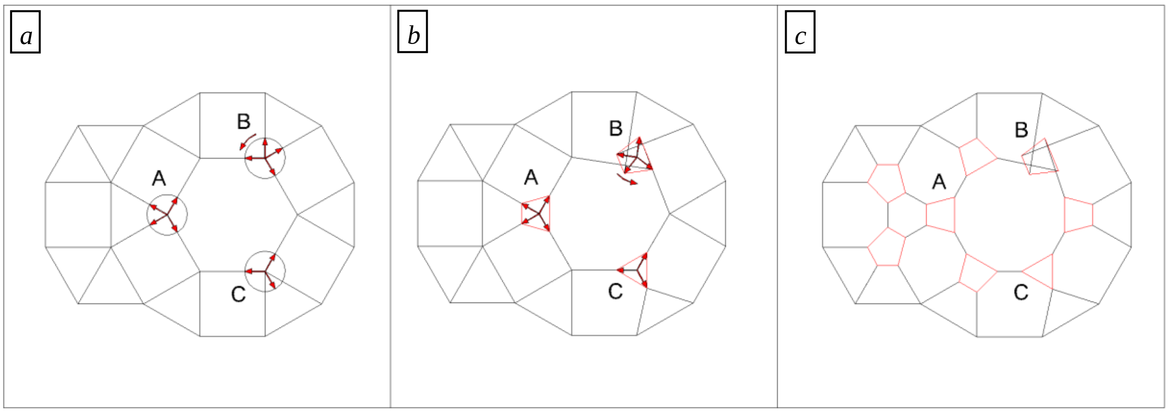

Figure 5. Natural rot-umbela manipulation of demi-regular tessellation with vertex valence 4 and 5 (u=v)

When talking about DLGs, rot-umbela manipulations can be applied to just one of the two layers or both, as well as to all the vertices of the grid or just some ones. Results are countless depending on the complexity of the DLG, and variety of new DLTGs is also remarkable.

For another example, let's take the DLG generated in Figure 4, let's apply a rot-umbela manipulation (u=3) on the bottom layer (*Figure 6*.a) and we'll obtain the tensegrity grid of Figure 6.b. Note that compressed elements are sets of tripods (class k=3) not touching each other.

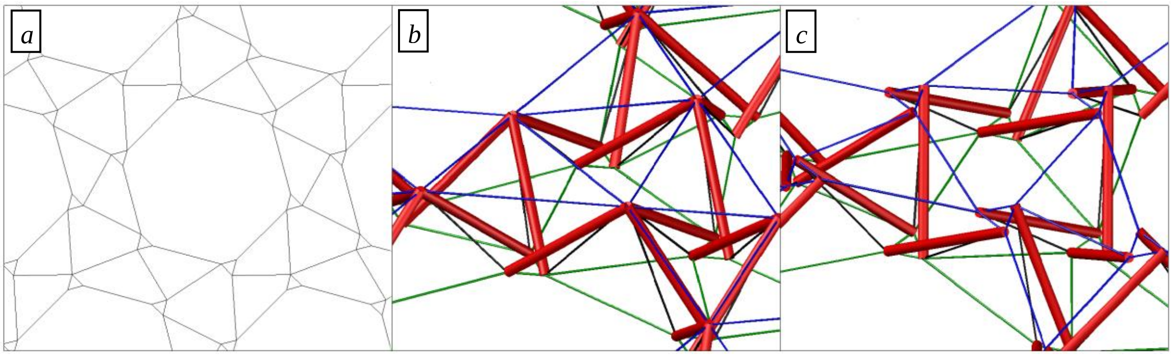

Figure 6. DLTG from 4,6,12: Rot-umbela manipulation (u=3) on lower chord. a) Bottom layer b) All layers c) Perspective.

If rot-umbela manipulation is applied also to top layer, a different layer would be attained (Figure 6.c.), a non-contiguous configuration (class k=1), composed of T-tripods similar to those used in grids of Figure 2.

It is remarkable that some configurations of DLTGs already known respond as the final result of a rot-umbela manipulation. It is the case of the hexagonal grid made of tripods of Figure 1, which is exactly the configuration obtained after a natural rot-umbela manipulation on bottom and top layer of DLG of Le Ricolais. This grid is easily obtained after Otero's rules from the regular mosaic 6^3, as illustrated in Figure 7.a and b.

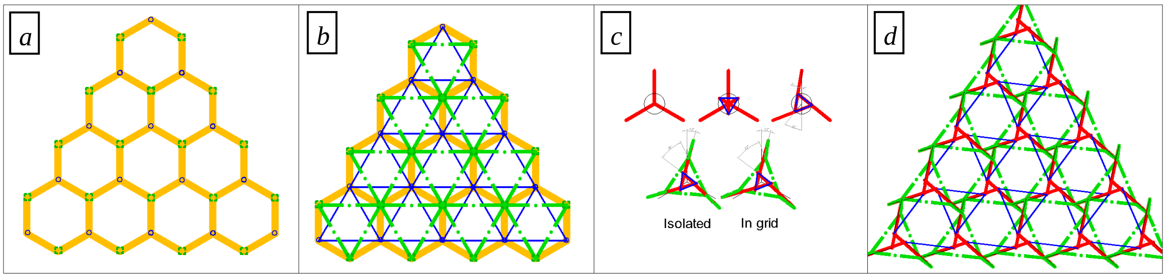

Figure 7. Obtaining Kono&Kunieda's DLTG from DLG of Le Ricolais with Rot-umbela Manipulation

As already mentioned, final geometry of the DLG is usually achieved by means of a rotation of the polygons opened around the vertices. It is widely known that in tensegrity prisms or pyramids, there is always a twist angle (a) between both bases, depending on the number of sides or struts of the system (n), following the formula $a = 90^\circ - 180^\circ / n$. In rot-umbela manipulations there is a twist angle as well, proportional to different factors that, because of its complexity, are not explained in this paper. In any case, as in every tensegrity structure, it is possible to correct that rotation by means of attaching additional tendons to the structure.

4.2. Other proposals

In addition to the new technique already described, research on other different ways of generating new DLTGs is being carried on, and will be presented in next communications. Some of them come from the variation of conventional DLGs, like the rot-umbela manipulations, while others come directly from the combination of actual tessellations or already existing tensegrity grids.

4.3. Analysis of the grid

In order to achieve the correct and stable configuration of the DLTGs proposed in precedent sections, it is essential to prove its stability and equilibrium by means of a study of the structure. Being this the case, a numerical process based on the force density method *[81],[82]* is considered for form-finding of the tensegrity grids. Literature with numerical examples is abundant, so we will avoid reiteration by keeping the extension of the present communication short.

Special attention has to be paid to edges of the grid and boundary conditions, due to their configuration it is critical for providing the stability and equilibrium to the whole system.

5. Future research

Analytical comparison between conventional and tensegrity double-layer grids are also being established depending on different factors: weight, resistance, deformation, clearance, economy of materials, etc. Obviously, important advantages and disadvantages are argued depending on the application of the final mesh. Thus, some conclusions about the possible application of the new DLTG are being arisen and will also be presented in the future.

6. Conclusions

Even though for the last years there have been several proposals for designing DLTGs, most of them have been obtained with a methodology based on composition, attaching tensegrity modules one to each other. As shown by Raducanu, there are other possibilities with interesting geometry and applications. A new approach is shown in this work, parting from tessellations that originate conventional DLG, which lower and/or upper chords are modified with rot-umbela manipulations. From this point, a new catalogue of possibilities is open for creating new arrangements of DLTGs.

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