Proceedings of the First Conference
Transformables 2013.In the Honor of Emilio Perez Piñero
18th-20th September 2013, School of Architecture Seville, Spain
EDITORIAL STARBOOKS. Felix Escrig and Jose Sanchez (eds.)

Comparison between new families of Double-Layer Tensegrity Grids

V. Gomez-Jauregui [1], C. Manchado [2], C. Otero [3]

[1] EGICAD Research Group, Assistant Lecturer, University of Cantabria, Santander, Spain, *valen.gomez.jauregui@unican.es*
[2] EGICAD Research Group, Assistant Lecturer, University of Cantabria, Santander, Spain, *manchadoc@unican.es*
[3] EGICAD Research Group, Associate Professor, University of Cantabria, Santander, Spain, *oteroc@unican.es*

Summary: Rot-Umbela manipulations permit conventional double-layer grids (DLG) to be transformed into tensegrity grids. By means of this method, two new tensegrity modules (Quastrut and Sixstrut) were already discovered. The aim of this work is to compare the behavior of the new family of Double-Layer Tensegrity Grids (DLTG) obtained by the juxtaposition of the Quastrut in some of its variations depending on enantiomorphic variants (e.g. monogyre Vs. racemic), orientations (e.g. 0º Vs. 90º) and configurations (e.g. open Vs. closed). It will be possible to determine which DLTG performs better taking into account their resistance, structural efficiency, deflection, etc. Furthermore, analysis of their mechanisms and states of self-stress could help to understand their structural characteristics better. Deployability will be revealed as one of the most challenging and interesting potentials of these DLTGs.

Keywords: *Tensegrity, Structure, Deployable, Double-Layer, Grid, Quastrut*

INTRODUCTION

Tensegrity systems are considered as self-stressed and auto-stable structures composed by isolated components in compression inside a net of continuous tension, in such a way that the compressed members (usually bars or struts) do not touch each other and the pre-stressed tensioned members (usually wires or even tensile membranes) delineate the system spatially [1].

It has been recently proved that the use of Rot-Umbela Manipulations, applied to Double-Layer Tensegrity Grids (DLTGs) produces a transformation to some other new and unknown, until now, tensegrity grids [2]. A closed observation of the new grids permits new kinds of tensegrity modules to be obtained, baptized as Quastruts and Sixstruts, integrated in the novel grids [3].

All the modules of the family are characterized for having some nodes with just two wires meeting at them, which simplifies the configuration of the nodes (and thus their costs) and makes any type of deployment of the module easier. A brief description of these components is provided, as well as some information about their static analysis, states of self-stress and internal mechanisms.

Nowadays, the principal use taken into consideration for Quastruts and Sixstruts is the generation of DLTGs (which is actually its origin), but these modules could also be implemented for the design of another kind of structures, like pedestrian bridges or light canopies.

The principal aim of this work is to compare the behavior of the new families of DLTGs obtained by the juxtaposition of the Quastrut in some of its variations depending on enantiomorphic variants (e.g. monogyre Vs. racemic), orientations (e.g. 0º Vs. 90º) and configurations (e.g. open Vs. closed). Besides, it would also be interesting to compare them with other DLTGs already existing and well known in the tensegrity field.

In such a way, and after analyzing their advantages and disadvantages, it will be possible to determine which DLTGs perform better taking into account their resistance, structural efficiency, deflection, etc.

ROT-UMBELA MANIPULATIONS

In the case of a grid or tessellation, a Rot-Umbela Manipulation is defined as a transformation of the vertex of a grid in such a way that it originates an "atomization" of a node, converting it to several nodes linked together and usually rotated around the original vertex. Final shape and rotation would be defined by the initial conditions imposed to geometry and state of self-stress applied to the structure. For any vertex of valence v, a new polygon of u sides could be generated around it, saying that it has an 'umbela valence' u. Vertex of Fig. 01 is processed with a 'natural' umbela valence (u=v=6) and a rotation of 120º.

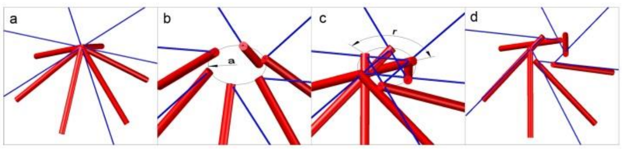

Fig. 01 Rot-Umbela manipulation in a grid.

GENERATION OF QUASTRUTS AND SIXSTRUTS

By means of applying a Rot-Umbela Manipulation to the DLTG 4^4-Be1-Te1 (nomenclature according to [4]), originally patented by Raducanu and Motro [5] under the name "2-way grid" and composed by expanders V22 (Fig. 02), it is possible to generate three new shapes inside the original grid.

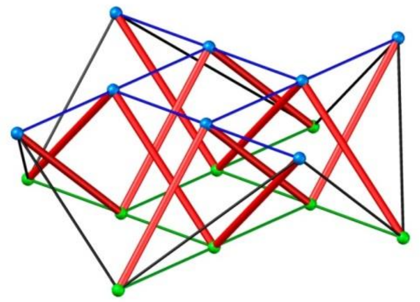

Fig. 02 DLTG 4^4-Be1-Te1 or "2-way grid"

These subsystems, when isolated, produce three innovative module configurations depending on the arrangement of the cables (struts always keep the same position for all types). Because they are composed of groups of four struts, they will be baptized as Quastruts (Fig. 03).

Quastrut-S: The first configuration of cables, in Fig. 03.a, is a module composed by four struts (1-7, 2-6, 3-8, 4-5) overlapping each other, an S-shape net of cables on the top layer (1-3, 3-2, 2-4, forming 90º between them, in dark blue lines), and another S-shape net of cables on the bottom layer (6-7, 7-5, 5-8, in clear green lines) rotated by 180º relative to the superior one. Four more wires in the periphery of the module (1-5, 2-8, 3-6, 4-7), close the sides of the module in the plan view. This module is super stable, as it is stated by the fact that its force density matrix is positive definite [6], having five mechanisms (m=5) and just one state of self-stress (s=1).

Quastrut-Z: The second variation, in Fig. 03.b, occurs when horizontal wires form a Z-shape (5-8, 8-6, 6-7 in bottom layer and 3-1, 1-4, 4-2 in top layer). Coordinates are the same as those of Quastrut-S, but the topology is different. However, this original configuration is not stable by itself, having four internal mechanisms (m=4) and no state of self-stress (s=0) capable to stiffen the structure. Thus, it cannot be considered a tensegrity structure on its own, but only when combined with other modules or stiffen by additional components.

Quastrut-S-Z: The third variation can be created when both configurations exposed above are mixed together. For instance, the bottom wires form a Z-shape while the top wires form an S-shape (or vice versa).

Another new tensegrity module can be obtained by applying a Rot-Umbela Manipulation to the DLTG 3^6-Be1-Te1 or "3-way grid". The result is the so-called Sixstrut (because of the six bars that it composes), another super stable tensegrity with just one state of self-stress (s=1) and six mechanisms (m=6). Some other new analogous structures have been discovered with different number of struts (Octastrut, Decastrut, Dodecastrut, etc.)

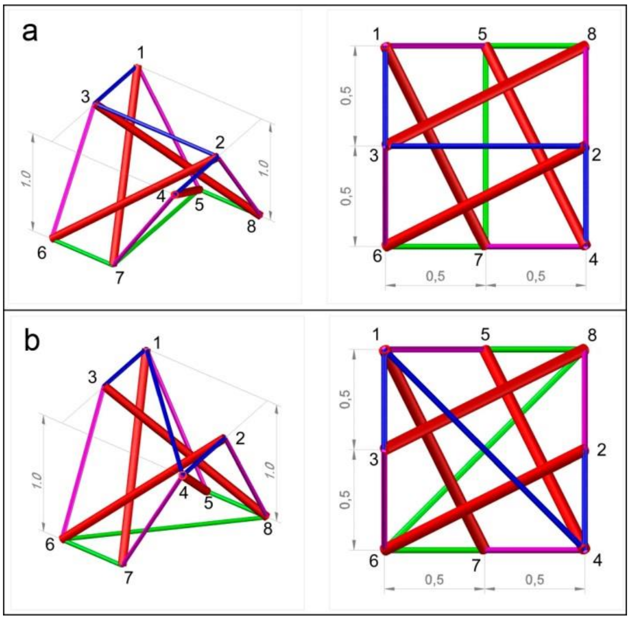

Fig. 03 a) Quastrut-S and b) Quastrut-Z.

COMPOSITION OF NEW DLTGS

It is easily conceivable to create a wide range catalogue of different DLTGs attending to the combinations of all of them. However, in this work only compositions made with the Quastrut-S and Quastrut-Z will be analyzed, as they are interesting enough to develop a significant case study.

All the modules exposed in the previous section, including the Quastruts that are being studied, are enantiomorphic, so it is possible to use a "monogyre" composition with either dextrorse or sinistrorse modules (d and s respectively in Fig. 04 and Fig. 05), or a "racemic" arrangement, i.e. using both dextrorotatory and levorotatory forms of the modules.

Enantiomers of Quastruts can also be rotated in the grid, aligning them at 0º or 90º, and thus conforming different grids by combining these two variations.

Even though there are multiple possibilities to combine the Quastruts and their variations, only four different possibilities for each one of them (Quastrut-S and Quastrut-Z) will be taken into account in order to keep the scope of the study manageable.

- Type 1: Monogyre, rotation 0º.
- Type 2: Monogyre, rotation 0º and 90º.
- Type 3: Racemic, rotation 0º
- Type 4: Racemic, rotation 0º and 90º

Classification of any tensegrity structure can be done depending on its class k (maximum number of struts concurring to the same joint). While types 1 and 4 are class 2, types 2 and 3 are class 4.

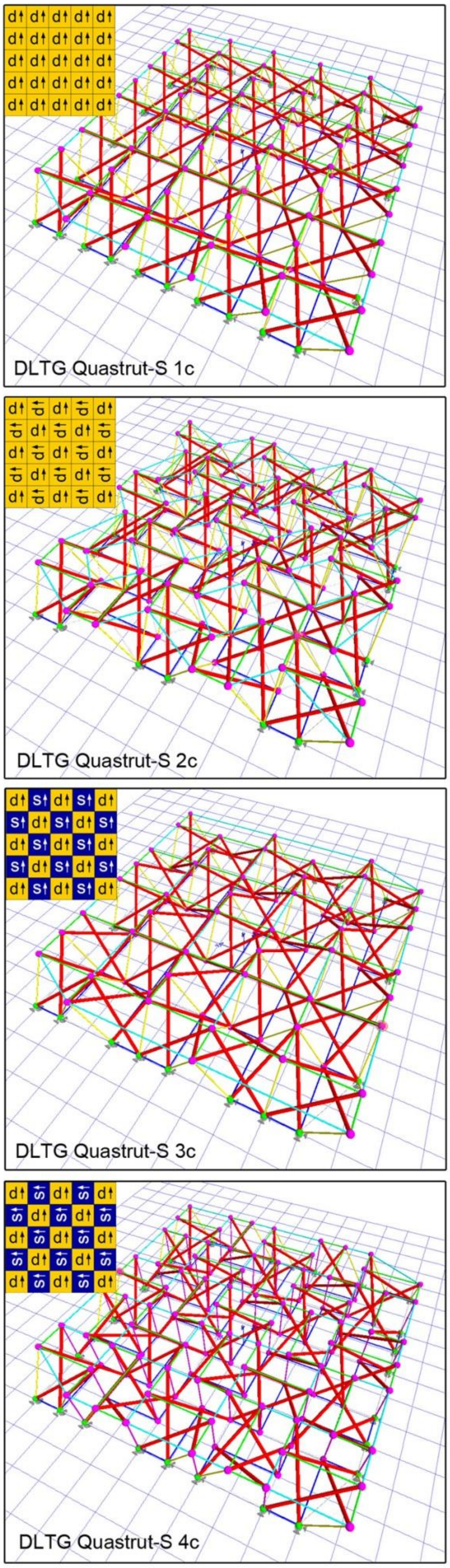

Fig. 04 DLTGs obtained with Quastruts-S

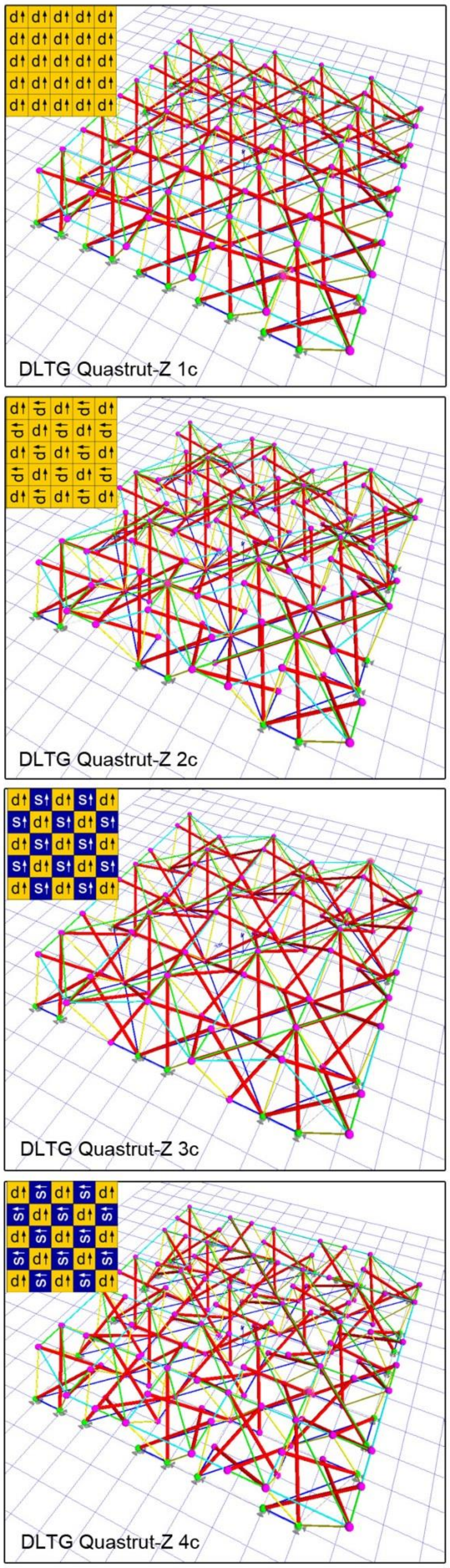

Fig. 05 DLTGs obtained with Quastruts-Z

The graphical representations of these grids are shown in Fig. 04 (for Quastruts-S) and Fig. 05 (for Quastruts-Z). For each of them, two variants have been considered:

- o) Open or Original one, just by juxtaposition of the modules.
- c) Closed or Covered one, by addition of cables to top and bottom layers to "fill the gaps" and reinforce the grid.

For the comparison of the grids, all of them have been designed flat and composed by 5x5 modules, each module being 2x2m in plan view and a total height of 1,5m. As a result, all the grids measure 10x10m with a depth of 1,5m.

DESIGN CRITERIA

Oncethe geometry of the DLTGs have been defined, as explained in the previous section, the boundary conditions must be fixed. Grids are simply supported at all the nodes lying on the boundaries of the lower layer.

After several trials, an initial set of conditions was established for accomplishing a feasible comparison of the behavior of the structures.

The material of all elements was steel (E=210000MPa, δ=7850kg/m^3), with different elastic limits for struts (f_y=355MPa) and cables (f_y=500MPa). Struts were defined with a round hollow structural section HSS60.3x6.9 (A=1008mm^2) and cables with a nominal diameter of 10mm (A=78.5mm^2).

Related to the load hypothesis, a simple combination of Ultimate Limited States (ULS) actions are considered: G + Q + S where (G) is the self-weight of the structure, (Q) the active loads and (S) the self-stress. As this study is a comparison between the behaviors of the DLTGs and not a real design process, at this stage no partial safety factors were considered.

Self-weight (G) is applied automatically by the program by defining section areas, lengths and densities of the different elements and considering a gravity acceleration of -1 in Z direction.

Application of active loads (Q) is distributed as a set of nodal masses among all the free nodes of the structure (those who are not supports). For this study, two uniform loads are applied, each one of 1kN/m2. The first one related to the typical roof live load and the second one responding to ground snow load. Because these conditions are certainly severe for such a structure, permanent loads of the covering roofing were considered negligible compared to them.

For the self-stress (S), a general and not optimized state of self-stress is applied to all the cables of the structure by introducing a pretension of 5000N (approximately 12.5% of their yield strength).

CALCULATIONS AND RESULTS

First of all, a study of the mechanisms and states of self-stress of each type of DLTG could help to understand their structural characteristics better. A numerical method to obtain the rank of the equilibrium matrix [7] applied to 4x4 DLTGs proves that the number of states of self-stress is significantly different in type S4 (s=8) and similar in the other cases (s=16 for S1 and s=17 for S2 and S3). The number of mechanisms is discordant enough for each type: 45, 53, 29 and 93 for DLTGs S1, S2, S3 and S4 respectively.

Static analyses of the structures have been carried out using the software ToyGL [8], a real time implementation of a discrete element method (mass-spring systems). This is an explicit dynamic nonlinear analysis, although for our purpose it was also used as a versatile method for the design and static analysis of tensegrity systems. It permits structures in real time to be created and modified, with a direct feedback on their behavior. It has been proved to be especially adequate for the design and calculation of tensegrity structures [8].

When working with this program, for automatically processing the input of the data (from an AutoCad file) and output of the results (to an Excel file), a customized set of routines have been developed by the authors of this contribution.

Weights of the grids are sensibly equivalent (maximum difference of 10%), as all of them have the same number of struts (100), which are the heaviest elements of the structure. The lightest DLTG is the S3 (2297kg) and the heaviest is the Z4cc (2562kg), with an average weight of 23,7kg/m^2.

Fig. 06 shows the behavior of all the DLTGs in terms of deflection (in cm) as well as maximum and minimum forces (in kN). As can be observed, there are three grids that collapse. This is not due to the lack of resistance to the applied loads, but to the lack of stability and self-equilibrium. The rest of the structures are able to support the external loads, but among them the minimum deformations correspond to the grid S1 (2.9cm) and S1c (2.8cm). For the other types these values are between approx. 6 and 40 cm.

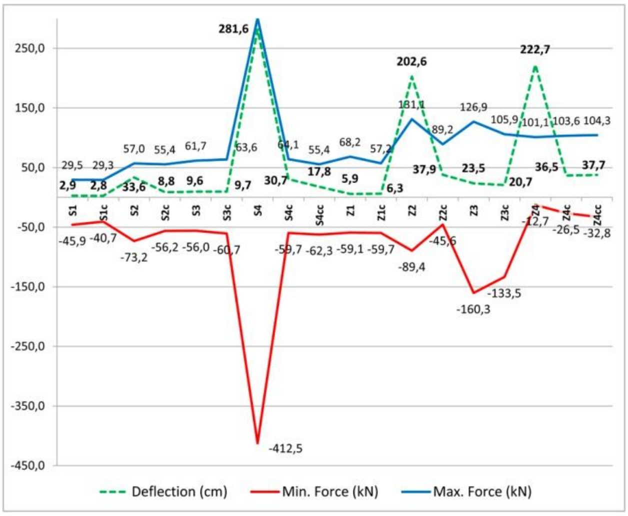

Fig. 06 Graphic of deflections and forces in the DLTGs.

Not considering collapsed structures, maximum forces (in tension) for the cables are between 29kN (again in DLTG S1 and S1c) and 131kN (for type Z2). Values higher than approx. 40kN would mean the plasticity of the cables, because with a diameter of 10mm and a cross-sectional area of 78.5mm^2 their stress would overpass the yield point (500Mpa).

Minimum forces (in compression) for the struts are between -26kN (for type Z4c) and -160kN (for type Z3). In this case, the accepted limit is not defined by the cross-sectional resistance to uniform compression, but by the buckling resistance of the struts. Calculations have been made according to the Eurocode 3 (Design of steel structures). Following the procedure exposed in section 6.3, for a HSS60.3x6.9 (A=1008mm^2, I=3.77E5 mm^4), partial factor γ_{M0}=1.05, f_y=355MPa and considering hinged-hinged connections, the maximum compression load could not be higher than 783kN (this value has not taken into account any safety factor as the main purpose of the study is not a final design but a general comparison between the different DLTGs).

Results of the calculations are also represented in Fig. 07, where numbers of collapsing members for each case is shown. It can be clearly observed that the number of struts that collapse under buckling is not significant. Apart from the grid S4, which has been clearly proved that collapses, only another three grids have some member failing under buckling: Z2, Z3 and Z3c (with 1, 4 and 3 struts respectively).

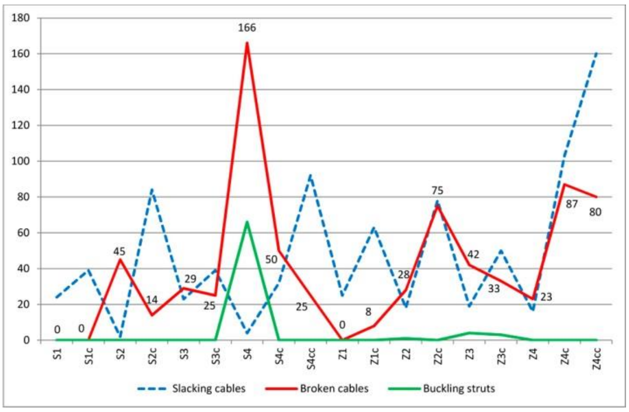

Fig. 07 Graphic of collapsing members of the DLTGs.

This figure also illustrates the number of “broken” cables that reach their yield limit (the term “broken” is not precise, but it serves to point out the most solicited members in tension). It is worth to remark that, again, grids S1, S1c and Z1 are the only ones with no collapsed cables. It can also be noted that, related to slacking cables (cables with null tension), even if their behavior is still good (between 20 and 40 slacking cables), there are some other DLTGs with similar or even better response. Slacking of cables is not an important problem from a mechanical point of view, but they can cause vibrations in case of wind and are not aesthetically pleasant.

It is straightforward to conclude that the best designs for this study case corresponds to DLTG S1 and S1c, composed by modules of Quastrut-S in juxtaposition, with no rotations and no reflections. Thus, in order to better understand the behavior of this structure, a deeper analysis of the grid S1c is going to be exposed.

Fig. 08 presents the distribution of forces of this structure, with the compressive forces (negative, downwards) and tensile forces (positive, upwards) of their different types of elements: struts (Web_Strut_Layer), top and bottom chords with the original cables of grid S1 (Upper_Lyr and Lower_Lyr), diagonal cables (Web_Wire_Lyr) and the additional cables included in the grid S1c for enhancing stiffness (Upper_Lyr_Triang and Lower_Lyr_Triang). As can be seen, these last additional groups of tension elements barely make any relevance in the overall disposition of forces of the structure, although obviously they contribute somehow to reduce the deflection and the maximum values of forces in struts and cables.

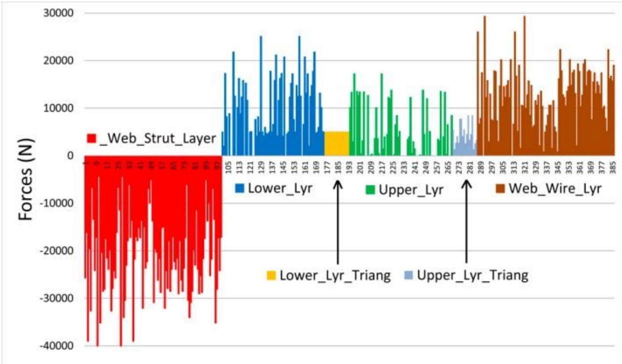

Fig. 08 Distribution of forces in the DLTG Quastrut-S 1c

It is be possible to optimize this DLTG by changing a few parameters. First of all, all the cables between supporting nodes can be erased because they do not bear any load but their own self-stress. Although increasing the tension of the upper cables seldom reduces the deflection, doubling the pre-stress in the diagonal wires reduces it almost 20%, which can reach up to 50% by imposing upon them a pretension of 20000N. It also reduces the number of slacking cables from 39 to 30. Another possibility, perhaps the most intuitive, is to raise the pre-stress of the lower chord, which reduces the deflection in 13% if the pretension reaches 10000N (decreasing the number of slacking cables and maximum/minimum forces in elements), or in 25% if it goes up to 20000N. A combination of both options, up to 20000N in each type of cable, would leave the total deflection in 1cm (64% reduction), 12 slacking cables (69% reduction) without a significant increase in the tension or compression carried by the members of the grid.

Optimization of the struts can be performed by changing the cross-section of those ones that bear low compression forces. 46 of them receive less than 26000N, so they are not in risk of buckling if their section is substituted by a HSS48.3x3.7 (A=480mm^2, I=1.22E5 mm^4). This change reduces the total weight by 25%, without any harm to the general behavior of the structure.

CONCLUSIONS

An analysis has been carried out to compare the behavior of a new family of DLTG obtained by the juxtaposition of the Quastrut in some of its variations. It is probably not a coincidence that the best behaviors correspond to those of the original DLTGs obtained directly from the Rot-Umbela Manipulations (DLTG Quastrut-S1 and DLTG Quastrut-Z1). However, what is interesting is the fact that these structures are class 2, when apparently a class 4 (grids type 2 and 3) should be stiffer and stronger. As expected, there is a certain influence of the number of states of self-stress and mechanisms in the overall response of these structures; grids with less states of self-stress and more mechanisms are more inclined to collapse, as happens with type S4.

In general, DLTG generated with Quastruts-S behave better than those composed by Quastruts-Z. This is more than probably due to the fact that the Quastrut-S is super stable by itself, while Quastrut-Z is not, and can only be in equilibrium when inserted in a bigger and more complex structure and supported properly.

It also looks clear that the improvement of any grid can be easily achieved by just adding a few cables on the top and bottom layers. However, real optimization of the grids is obtained by changing the initial pre-stress of the cables and reducing the cross-section areas of the least loaded struts. As a result, it is possible to obtain a light structure of 17.6 kg/m2, composed by juxtaposition of Quastruts-S, with no rotation or reflection, with an acceptable resistance to self-weight and external active loads.

FURTHER RESEARCH

A deeper study of the self-stress of each grid, the choice of its level, the design of the elements (cross-section areas, types of section, materials, etc.) will be necessary to optimize the design of the DLTGs. Besides, it would also be interesting to compare them under exactly the same conditions as other DLTGs already existing and well known in the tensegrity field.

It is not the intention of the present work to analyze in depth, but yes to mention, an interesting performance of the Quastruts: deployability. Physical models prove that their singular topology and geometry may lead to the consideration of several ways of folding and unfolding the grids composed by these modules.

A first way of folding Quastrut-S is shown in the Fig. 09.b, and even if it cannot be appreciated in pictures, release of the element that fixes the module in that flat configuration makes the module come back to its original unfolded shape (Fig. 09.a) automatically thanks to the elastic behavior of the tendons. A second way of folding is shown in Fig. 09.c and d, where the first one is the step in which the edges of the bottom (i.e. 6 and 8) and top cables (i.e. 1 and 4) that have an S-shape are detached from the struts, whose edges (i.e. 1', 4', 6' and 8') run through those cables until they approach the adjacent vertices of the other struts edges (i.e. 2, 3, 5 and 7). Second step is clearly illustrated in Fig. 09.d.

These characteristics make an in depth analysis of the possible deployability and foldability of the DLTGs exposed in this work feasible.

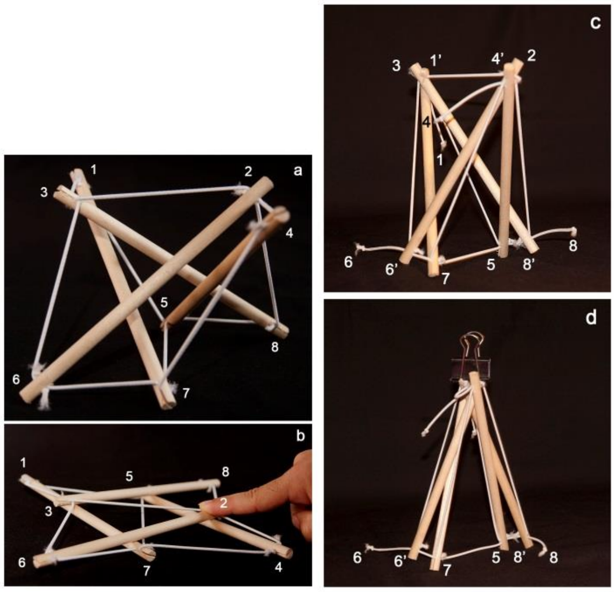

Fig. 09 a) Quastrut-S in unfolded position. b) Folding by elasticity, pushing down. c) Folding by disconnection of edges of the S and approaching vertices, first step. d) Second step.

References

[1] Gomez-Jauregui, V., Tensegrity structures and their application to architecture. Santander: Universidad de Cantabria. Servicio de Publicaciones, 2010.

[2] Gomez-Jauregui, V., Arias, R., Otero, C. & Manchado, C., Novel Technique for Obtaining Double-Layer Tensegrity Grids, International Journal of Space Structures, vol. 27, Special Issue 2–3, pp. 155–166, Jun. 2012.

[3] Gomez-Jauregui, V., Otero, C., Arias, R. & Manchado, C., Innovative Families of Double-Layer Tensegrity Grids: Quastruts and Sixstruts, Journal of Structural Engineering ASCE, Sep. 2012.

[4] Gomez-Jauregui, V., Otero, C., Arias, R. & Manchado, C., Generation and Nomenclature of Tessellations and Double-Layer Grids, Journal of Structural Engineering ASCE, vol. 138, no. 7, pp. 843–852, Jul. 2012.

[5] Raducanu, V. & Motro, R., Stable self-balancing system for building component, Patent WO02081832, granted 09-Apr-2001.

[6] Connelly, R., Tensegrity Structures: Why are They Stable?, in Rigidity theory and applications. MF Thorpe and PM Duxbury (Eds.), Kluwer/Plenum Publishers, pp. 47–54, 1999.

[7] Pellegrino, S. & Calladine, C.R., Matrix analysis of statically and kinematically indeterminate frameworks, International Journal of Solids and Structures, vol. 22, no. 4, pp. 409–428, 1986.

[8] Averseng, J., Quirant, J. & Dubé, J.-F., Interactive design and dynamic analysis of tensegrity systems, International Journal of Space Structures, vol. 27, Special Issue 2–3, Jun 2012.